MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
ADVANCED LEVEL

MAY 2016

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	I
DATE:	**7th May 2016**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. ABCD is a square of side a. Forces of magnitudes 4, 3, 2, 1, P, Q act along $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{AD}, \overrightarrow{AC}, \overrightarrow{BD}$ respectively.

   (i) Find P and Q if the resultant acts along AB.

   (ii) Find the magnitude of the resultant if its line of action passes through B and is parallel to $\overrightarrow{AC}$.

[6, 4 marks]

2. A and B are two points on level ground, 60 m apart. A particle is projected from A towards B with initial velocity 30ms^{-1} at $45°$ to the horizontal. At the same instant, a particle is projected from B towards A with the same speed at $45°$ to the horizontal. It can be assumed that initially, at time $t = 0$, A is at the origin, and B lies on the positive side of the x-axis. Find:

   (i) the displacement of the two particles as a function of t;

   (ii) the time t at which the particles collide;

   (iii) the height above the level of AB at which they collide;

   (iv) the Cartesian equation of the path taken by A.

[4, 2, 2, 2 marks]

3. A uniform lamina ABCD is in the form of a square of side $2a$. E and F are the midpoints of BC and CD. The lamina is then folded about the line EF, so that vertex C now coincides with the centre of the square.

   (i) Find the distance of the centroid of the resulting lamina from AB and AD.

   (ii) The lamina is now placed vertically with its side BE in contact with a plane inclined at an angle α to the horizontal. BE lies on a line of greatest slope of the plane, with B at a higher level than E. Find the largest value of α if the lamina does not topple over.

[7, 3 marks]

4. A uniform sphere of radius a rests against a vertical wall supported by an inelastic string of length $2a$ fixed to a point on its surface and to a point on the wall.

   (i) If the wall is smoooth, find the inclination of the string to the vertical.

   (ii) If the wall is very rough, and the sphere is on the point of slipping down the wall when the string is at $30°$ to the vertical, find the coefficient of friction between the sphere and the wall.

[5, 5 marks]

5. A truck of mass 10,000 kg has a maximum speed of 24 km/hr up a slope of 1 in 10 against a resistance of 1200 N.

   (i) Find the effective power of the engine in kilowatts.

   (ii) If the resistance varies as the square of the speed, find the maximum speed on level ground to the nearest km/hr.

[5, 5 marks]

6. A light triangular framework consists of three light rods AB, BC and CA of length $13a$, $5a$ and $12a$ respectively, smoothly jointed together at their ends to form a triangle ABC with a right angle at C.

   The framework, which lies in a vertical plane, rests on supports at A and B, with AB horizontal, and with C vertically above AB. The system carries a load W at C.

   Find the reaction at the supports, and the forces in the rods, indicating whether they are in tension or compression.

[10 marks]

7. A small smooth sphere moves on a horizontal table and strikes an identical sphere lying at rest on the table at a distance d from a vertical wall, the impact being along the line of centres and perpendicular to the wall. Prove that the next impact between the spheres will take place at a distance

$$\frac{2de^2}{1+e^2}$$

   from the wall, where e is the coefficient of restitution for all impacts involved.

[10 marks]

8. Two light inelastic strings AP and BP connect a particle P to fixed points A and B. The point B is vertically above A and AB = AP = l and BP = $l\sqrt{3}$. The particle P moves in a horizontal circle with constant speed. The least angular speed of P for both strings to be taut is ω_o.

   (i) If both strings are taut, find the angle between the strings and obtain an expression for ω_o in terms of g and l.

   (ii) When P has an angular velocity $\omega_1 (> \omega_o)$ the tensions in the strings are equal. Show that $\omega_1^2 = 2g/(l\sqrt{3})$.

[2, 5; 3 marks]

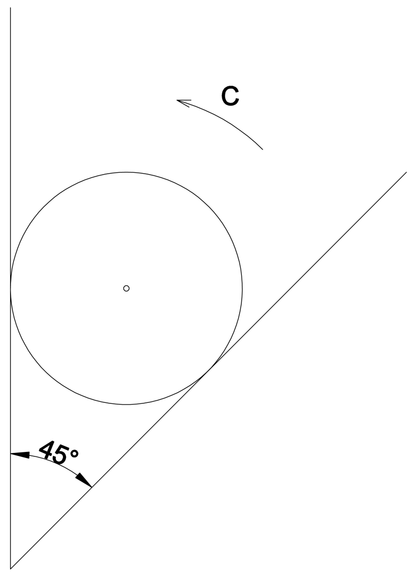
9. Two identical springs AB and BC of natural length a and modulus of elasticity $2mg$ are fastened together at B. Their other ends A and C are fixed to two points $4a$ apart in a vertical line, with C vertically above A. A particle of mass m is attached at B. The system is in equilibrium.

   (i) Find the length of CB at equilibrium.

   (ii) The particle is displaced slightly downwards, so that it starts making vertical oscillations along AC. Show that the motion is simple harmonic and find its frequency.

[5, 5 marks]

10. A uniform cylinder of radius a and weight W rests with its axis horizontal and its curved surface in contact with a rough vertical wall and with a rough plane inclined at 45° to the horizontal. The coefficient of friction between the cylinder and the wall, and the cylinder and the plane is μ.



Assuming that friction is limiting at both the wall and the plane, show that the greatest couple, C, that will not rotate the cylinder is given by

$$\frac{\mu a W}{1+\mu^2}(\sqrt{2}+1+\mu).$$

[10 marks]

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MATRICULATION EXAMINATION
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MAY 2016

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	II
DATE:	**9th May 2016**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Answer **SEVEN** questions. In all there are 10 questions each carrying 15 marks.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, **i**, **j**, **k** are unit vectors along the x-, y- and z-axes of a Cartesian coordinate system.

(Take $g = 10$ ms^{-2}).

1. A light uniform beam ABC has length $2a$ and midpoint B. It is clamped horizontally at A, and carries a load W at B, and another load W at C. This cantilever system is in equilibrium.

   (i) Find the reaction and couple at the clamped end of the beam.

   (ii) Find the shearing force and bending moment at any point along the beam.

   (iii) Draw a sketch of the shearing force and bending moment.

   (iv) Find the maximum bending moment and the point on the beam where it occurs.

[2, 8, 4, 1 marks]

2. A particle of mass m kg is acted upon by a constant force of $21m$ Newtons in the direction of the vector $3\mathbf{i} + 2\mathbf{j} - 6\mathbf{k}$.

   Initially, at $t = 0$ seconds, the particle is at the origin moving with speed 18 ms^{-1} in the direction of the vector $7\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$.

   (i) Find the position vector of the particle at time t for $t > 0$. Hence find the position of the particle at $t = 4$ s.

   (ii) Using the scalar product of two vectors, find the work done by the force during these 4 seconds.

   (iii) Find the increase in kinetic energy over this time range, and explain why it should be equal to the result in (ii).

[8; 4; 2, 1 marks]

3. The position vectors of the vertices A, B, C of a triangle are respectively

$$4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} \qquad 2\mathbf{i} + \mathbf{j} + 4\mathbf{k} \qquad -3\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}.$$

   (i) Find the position vector of G, the centroid of the triangle.

   (ii) Forces $3\overrightarrow{AB}$ and $2\overrightarrow{AC}$ act along AB and AC. Find the magnitude of their resultant and the position vector of the point P in which the line of action of this resultant meets BC.

   (iii) Finding the vector product of $\overrightarrow{GA}$ and $\overrightarrow{AP}$, or otherwise, show that the area of triangle GAP is $\sqrt{5}/2$. Hence find the moment of the resultant about an axis through G perpendicular to the plane ABC.

[1; 1, 6; 4, 3 marks]

4. Two identical smooth spheres are moving on a horizontal table with velocity vectors $3\mathbf{i} + 4\mathbf{j}$ and $-\mathbf{i} + \mathbf{j}$ and collide when the line joining their centres is parallel to the vector $\mathbf{i}$. The coefficient of restitution between the spheres is $\frac{1}{2}$.

   (i) Find the velocity vectors of the spheres after impact.

   (ii) Find the relative velocities of the two spheres before and after impact, and find the ratio of their magnitudes.

   (iii) If at the instant of impact, the centres of the spheres are 2 units of distance apart, find the distance between their centres 1 unit of time later.

[7, 4, 4 marks]

5. A vehicle of mass m is being driven by an engine with power output, P, where P is a constant. The vehicle starts from rest and moves along a horizontal road against a resistance, Rv, where R is a constant.

   (i) Using Newton's Second Law of Motion, set up a differential equation relating the speed v of the vehicle to t, the time elapsed in seconds.

   (ii) Integrate this differential equation to find v in terms of t.

   (iii) Show that the maximum speed of the car is $\sqrt{\dfrac{P}{R}}$.

[4, 10, 1 marks]

6. Two uniform rods, AB and BC are of the same length and weigh $3W$ and W respectively. They are smoothly jointed at B and stand in a vertical plane with A and C on a rough horizontal plane. The coefficient of friction between each rod and the plane is $\frac{2}{3}$. Equilibrium is about to be broken by one of the rods slipping on the plane.

   (i) Find which rod will slip and calculate the angle which each rod makes with the plane.

   (ii) Calculate the reaction at the hinge B in magnitude and direction.

[10, 5 marks]

7. A heavy particle is projected horizontally with speed u from the lowest point on the inside of a hollow smooth sphere of internal radius a.

   (i) Show that the least value of u for the particle to complete a vertical circle is $\sqrt{5ga}$.

   (ii) The particle is projected with this velocity and hits a rubber peg after travelling a distance $\frac{3}{2}\pi a$, the coefficient of restitution between the peg and the particle being $\frac{1}{2}$. Find the velocity of the particle just after impact. Hence calculate the vertical height of the particle above the point of projection at the moment when it leaves the surface of the sphere.

[8; 2, 5 marks]

8. A uniform circular disc has mass m, radius a and centre O. P is a point on the disc so that OP $= x$.

   (i) Find by integration the moment of inertia of the disc about an axis passing through O and perpendicular to the disc. Deduce by the theorem of parallel axis, the moment of inertia about an axis passing through P and perpendicular to the disc.

   (ii) The disc performs small oscillations about a fixed smooth horizontal axis passing through P and perpendicular to the disc. Find the periodic time of these oscillations.

   (iii) Find the value of x for which the periodic time of these oscillations is a minimum.

[6, 6, 3 marks]

9. A uniform rod AB of mass m and length $2a$ has a particle of mass m attached at the end B. It is free to rotate about a fixed horizontal axis through A.

   (i) Write down the moment of inertia of the system about this axis.

   (ii) The system is held in a vertical position with B vertically above A, and released from rest. Find the angular velocity of the rod when it is in its lowest postion.

   (iii) When the rod hangs vertically at rest, a horizontal impulse J at right angles to the axis is applied at B. By taking moments about A, and using the result in (ii), or otherwise, show that the rod makes a complete revolution if $J \geq 4m\sqrt{ga}$.

[2, 8, 5 marks]

10. A uniform sphere of mass m and radius a rolls without slipping down a rough plane inclined at an angle α to the horizontal. The coefficient of friction is μ.

   (i) Find the acceleration of the sphere down the plane.

   (ii) Find the least value of μ if the sphere is to roll without slipping down the plane.

[10, 5 marks]