MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
ADVANCED LEVEL

MAY 2017

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	I
DATE:	**12th May 2017**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ $\mathbf{ms}^{-2}$).

1. A uniform rod AB has mass 10 kg and length 2 m. C is a point on the rod, such that AC = 1.5 m. The rod rests with A on rough horizontal ground, and with C resting on a smooth peg 0.75 m above the ground. Under these conditions, the system is in limiting equilibrium.

   Find the reactions at A and C, and the coefficient of friction at A.

   (Total: 10 marks)

2. A ball is projected from the top of a vertical cliff, with speed V ms^{-1} at 30° above the horizontal. The top of the cliff is 50 m above the level of the sea. The ball strikes the sea at a horizontal distance of 80 m from the base of the cliff. Find:

   (i) the Cartesian equation of the path of the ball in terms of V; (4)

   (ii) the velocity of projection and the time of flight; (2, 2)

   (iii) the maximum height reached by the ball above the point of projection. (2)

(Total: 10 marks)

3. A uniform disc has centre O, diameter AOB, and radius 2 cm. D is a point on the circumference such that $\angle AOD = 90°$. A circular portion with diameter OB is removed from the disc.

   (i) Find the position of the centroid of the resulting lamina. (7)

   (ii) The lamina is now suspended smoothly from D. Find the angle which AO makes with the downward vertical. (3)

(Total: 10 marks)

4. A smooth sphere P of mass 3 kg moving at a speed of 5 ms^{-1} collides directly with a stationary smooth sphere Q of mass 2 kg. This sphere then hits a wall at right angles, and bounces back to collide directly with P for a second time. It can be assumed that all collisions are direct, and that the coefficient of restitution for all collisions is 1/2. Find:

   (i) the velocities of the spheres just after the first collision; (4)

   (ii) the velocity of Q just after impact with the wall; (1)

   (iii) the velocities of the spheres just after they collide for the second time; (4)

   (iv) the impulse of each sphere on the other in the second collision. (1)

(Total: 10 marks)

5. A car of mass 1000 kg travels on a level road at a constant speed of 36 km/hr with its engine working at 15 kW. Throughout this question it can be assumed that the total resistance to motion is proportional to the speed of the car. Find:

   (i) the resistance to motion as a function of the speed; (3)

   (ii) the maximum speed of the car on the same road if the engine is working at its maximum power of 25 kW; (3)

   (iii) the acceleration of the car when it is travelling at 18 km/hr up an incline of $\sin^{-1}(1/10)$ with the engine working at its maximum power of 25 kW. (4)

(Total: 10 marks)

6. A light framework consists of five rods, four of which form the edges of a square ABCD, the fifth rod acting as the diagonal AC of the square. All rods are smoothly jointed at their ends to form the framework.

   The framework is suspended smoothly from A, and carries loads W at B, $2W$ at C, and W at D. At equilibrium, AC is vertical, and the system is symmetric about AC.

   Find the reaction at A, and the forces in the rods, indicating whether they are in tension or compression.

(Total: 10 marks)

7. A pendulum bob, P, of mass 1.2 kg, hangs at one end of a light, inextensible string which passes through a smooth hole in a table at a point O. The length of OP is 0.7 m. The other end of the string is attached to a particle, Q, of mass 5.2 kg, which is resting on the rough horizontal surface of the table. The coefficient of friction between Q and the table is 0.25. The bob, P, is made to move as a conical pendulum below O.

   Find the maximum angular speed at which the bob P can move without making Q slip.

(Total: 10 marks)

8. A heavy uniform rod AB of weight w_1 and length $4l$ is hung from a point O by two equal strings OA and OB, which are attached respectively to the ends A and B of the rod. A weight w_2 is attached to the point C on the rod, where AC$= 3l$.

   By taking moments about A and B respectively, or otherwise, find the ratio of the tensions in the strings in terms of w_1 and w_2.

(Total: 10 marks)

9. A man can push his car of mass 1250 kg with a force F Newtons in the direction of motion such that $F = 250 - 4x$, where x metres is the distance moved from rest along the road. The man pushes the car from rest for a distance of 50 metres. Find:

   (i) the work done by the man; (4)

   (ii) the speed attained by the car on a horizontal road, if there are no resistive forces acting; (3)

   (iii) the speed attained by the car on a horizontal road, if there is a constant resistive force of 40 N. (3)

(Total 10 marks)

10. An elastic string has natural length a and modulus of elasticity mg. Particles of masses m and $2m$ are attached to its ends. The particles are held at rest at a distance $3a$ apart on a smooth horizontal surface, and are then released.

    Find the speeds of the particles at the moment when the string goes slack, and find the point where they collide.

(Total: 10 marks)

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MATRICULATION EXAMINATION
ADVANCED LEVEL

MAY 2017

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	II
DATE:	**13th May 2017**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Answer **SEVEN** questions. In all there are 10 questions each carrying 15 marks.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, **i**, **j**, **k** are unit vectors along the x-, y- and z-axes of a Cartesian coordinate system.

(Take $g = 10$ $\mathbf{ms}^{-2}$).

1. A light rigid beam of length 6 m is supported horizontally at two points 1 m from each end, and carries a uniformly distributed load of 400 N/m over its whole length. This structural system is in equilibrium.

   (i) Find the normal reactions at the supports of the beam. (2)

   (ii) Find the shearing force and bending moment at any point along the beam. (9)

   (iii) Draw a sketch of the shearing force and bending moment, and deduce where the bending moment is largest. (4)

(Total: 15 marks)

2. The forces $\mathbf{F}_1 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ N and $\mathbf{F}_2 = -\mathbf{i} + 2\mathbf{k}$ N act on a rigid body at the points whose position vectors relative to the origin O are $\mathbf{r}_1 = \mathbf{j} + \mathbf{k}$ and $\mathbf{r}_2 = 4\mathbf{i} + \mathbf{j} - \mathbf{k}$ respectively. A third force $\mathbf{F}_3 = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ acting at the point with position vector $\mathbf{r}_3 = \alpha\mathbf{i} + \beta\mathbf{j} + 5\mathbf{k}$, is then added so that $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = \mathbf{0}$, and the sum of the moments of the three forces about O is equal to $-7\mathbf{i} + 9\mathbf{j} - 6\mathbf{k}$.

   (i) Find $\mathbf{F}_3$ and $\mathbf{r}_3$. (9)

   (ii) A fourth force $\mathbf{F}_4$, acting at the point $3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, is then added to the above system so that the sum of the moments of the four forces about O is zero.

   Find a possible value of $\mathbf{F}_4$, and state with reasons whether the system is in equilibrium. (6)

(Total: 15 marks)

3. Two birds, A and B, are initially at points with position vectors $5\mathbf{i} + 8\mathbf{j} + 12\mathbf{k}$ m and $2\mathbf{i} - 4\mathbf{j} + 15\mathbf{k}$ m respectively. They are flying with constant velocities $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ ms^{-1} and $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ ms^{-1} respectively.

   Find the time they are closest together, and the distance between them at this instant.

(Total: 15 marks)

4. A pendulum bob of mass m is fastened to one end of a string of length a whose other end is fixed at a point O. The bob is at rest in its lowest position when it is set in motion with initial speed $\sqrt{7ga/2}$. As it swings upwards, the string meets a small peg, P, on the same level as O. The string then wraps around P.

   If the bob is to make a complete revolution about P, find the smallest possible distance between P and O.

(Total: 15 marks)

5. An object of mass 5 kg is dropped from a great height. As it falls, it experiences a resistance of kv N, where v is the speed of the object, and k is a constant.

   (i) Using Newton's Second Law of Motion, set up a differential equation relating the speed v of the object to t, the time elapsed in seconds. (3)

   (ii) Integrate this differential equation to find v in terms of t. (10)

   (iii) Show that the velocity of the object approaches a constant for large values of t. (1)

   (iv) Find the value of k if the limiting velocity is 60 ms^{-1}. (1)

(Total: 15 marks)

6. The weight of an object below ground level is directed towards the centre of the Earth, and has a magnitude of kr, where r is its distance from the centre of the Earth, and k is a constant.

   A tunnel, in the form of a straight tube, is constructed joining two points on the surface of the Earth. An object is released from rest at one end of the tunnel.

   (i) Show that the object performs simple harmonic motion about the centre of the tunnel. (9)

   (ii) If the radius of the Earth is 6.4×10^6 m, find the value of k. (3)

   (iii) Find the time taken by the object to reach the other end of the tunnel. (3)

(Total: 15 marks)

7. ABCD is a rhombus made of freely jointed rods, each of length a and weight W. A and C are connected by means of a spring of natural length a and modulus of elasticity λ. The structure is supported from a fixed support at A. The angle between AB and the vertical is denoted by θ.

   (i) Find the potential energy, $P(\theta)$ of the system in terms of θ. (8)

   (ii) By finding $\dfrac{dP}{d\theta}$ and equating it to zero, find the values of θ for which the system is in equilibrium.

   Show that if $\lambda < 2W$, the only position of equilibrium is $\theta = 0$. (4)

   (iii) Find $\dfrac{d^2P}{d\theta^2}$. Show that if $\lambda < 2W$, then $\dfrac{d^2P}{d\theta^2} > 0$ for $\theta = 0$. This implies that this position of equilibrium is stable. (3)

(Total: 15 marks)

8. A uniform circular disc has mass m, radius a and centre O. P is a point on the circumference of the disc.

   (i) Find by integration the moment of inertia of the disc about an axis passing through O and perpendicular to the disc. Deduce by the theorem of parallel axis, the moment of inertia about an axis passing through P and perpendicular to the disc. (5)

   (ii) The disc can rotate freely about a fixed smooth horizontal axis passing through P and perpendicular to the disc. The disc is held with PO horizontal, and is then released from rest. Find the angular velocity and the angular acceleration of the disc in terms of θ, the angle between PO and the horizontal. (6)

   (iii) Find the vertical component of the reaction at P when PO is vertical. (4)

(Total: 15 marks)

9. A uniform rod OB of mass m and length $2a$ can rotate in a vertical plane about a fixed smooth horizontal axis passing through O and perpendicular to the plane of rotation.

   The rod is held with B vertically above O, and released from rest. In the subsequent motion, when the rod is in the vertical position again (with B below O), the rod hits a fixed peg at its end B, and the rod bounces back, coming to instantaneous rest with B on the same horizontal level as O.

   (i) Write down the moment of inertia of the system about this axis. (2)

   (ii) Find the angular velocity of the rod just before and just after it collides with the peg. (9)

   (iii) By taking moments about O, find the impulse exerted by the peg on the rod. (4)

(Total: 15 marks)

10. A uniform solid cylinder of radius a, is rolling without slipping up a rough plane inclined at an angle α to the horizontal. At a given instant, the angular velocity of the cylinder is ω.

    Show that it will travel a further distance of $\dfrac{3a^2\omega^2}{4g\sin\alpha}$ up the slope before coming to instantaneous rest.

(Total: 15 marks)