MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
ADVANCED LEVEL

SEPTEMBER 2017

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	I
DATE:	**4th September 2017**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. Darts are thrown at a vertical dartboard. The trajectory of each dart thrown is in a vertical plane. One dart is projected horizontally from a point 1.7 m above a horizontal floor and 3 m away from the dartboard. The dart strikes the board at a point which is 1.6 m above the floor. Find:

   (a) the time taken by the dart to reach the dartboard; (4)

   (b) the speed with which the dart was projected; (2)

   (c) the angle which the trajectory of the dart makes with the vertical when it strikes the dartboard; (2)

   (d) the Cartesian equation of the path of the dart. (2)

(Total: 10 marks)

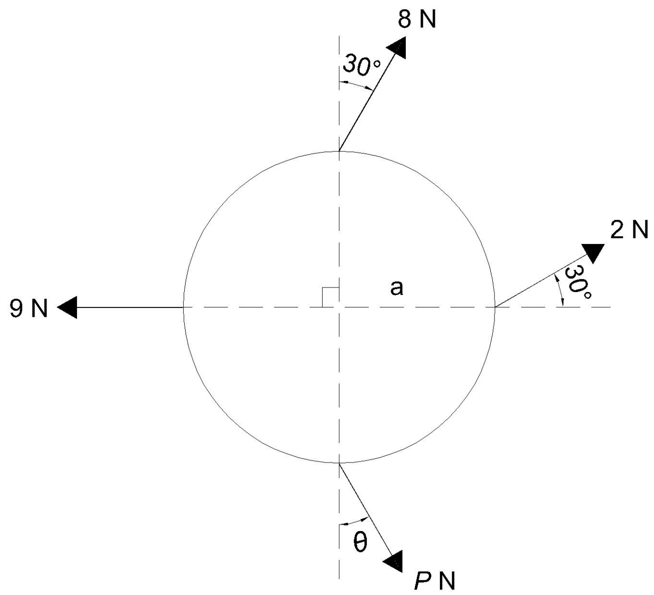
2. A cool-box consists of a cylinder open at one end, together with a hollow hemispherical cap which is attached to the open end of the cylinder. The cylinder and cap both have an exterior radius of 30 cm, and an internal radius of 20 cm. The exterior height of the cylinder is 80 cm, its interior height is 70 cm, and the thickness of its base is 10 cm. The cool-box is symmetrical about its central axis, and is made from a material of uniform density.

   (a) Find the height of the centre of mass of the empty cool-box above its base. For this part, you can assume that the centre of mass of a uniform hemisphere of radius r is at a distance of $3r/8$ from its centre. (7)

   (b) The cool-box is now suspended from a point Q on the rim of its base. Find the angle which the diameter through Q makes with the downward vertical. (3)

(Total: 10 marks)

3. The diagram shows a circular lamina of radius a. Forces of 9 N, 8 N, 2 N and P N act at the ends of two perpendicular diameters, as shown in the diagram.



   (a) Show that the system cannot be in equilibrium. (5)

   (b) Show that if the system reduces to a couple, the magnitude of the couple is $(2-\sqrt{3})a$. (3)

   (c) If the value of P is 10 and the system reduces to a single force through the centre of the circle, find the value of θ. (2)

(Total: 10 marks)

4. A block of mass m rests on a rough plane inclined at an angle α to the horizontal.

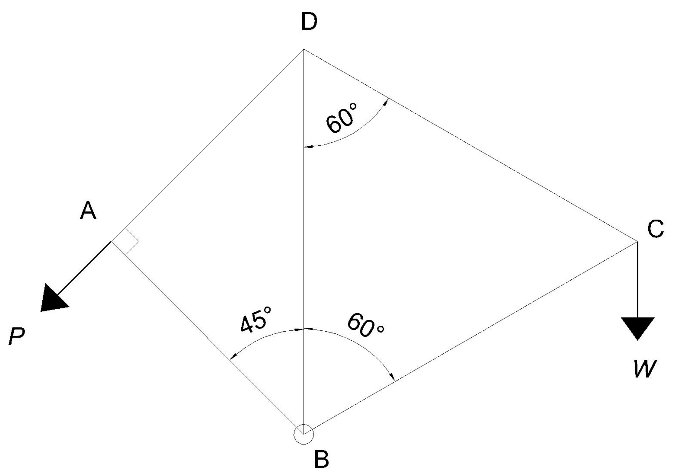
   A force P_1 acting up the plane causes the block to be on the point of moving up the plane.

   A force P_2 acting down the plane causes the block to be on the point of moving down the plane.

   Show that $P_1 - P_2$ is independent of the coefficient of friction between the block and the plane.

(Total: 10 marks)

5. A light framework consists of four light inextensible rods, AB, BC, CD and DA smoothly jointed together to form a quadrilateral ABCD, which is held in shape by a fifth rod BD, which is a diagonal of the quadrilateral. The triangle CDB is equilateral, whilst triangle ABD has AB = AD, with angle A = 1 right angle. The system is smoothly hinged at B and is held in equilibrium in a vertical plane with D vertically above B, by means of a vertical load W at C and a force P acting at A in the direction from D to A.



Find in terms of W:

(a) the value of P and the reaction at the hinge B; (5)

(b) the forces acting in the rods, indicating whether they are in tension or in compression. (5)

(Total: 10 marks)

6. A particle, A, of mass 3 kg is placed on a rough, horizontal turntable and is connected to its centre, O, by a light, inextensible string, OA, of length 0.8 m. The coefficient of friction between the particle and the turntable is 0.4. The turntable is made to rotate at a uniform speed.

If the tension in the string is 50 N, find the angular speed of the turntable.

(Total: 10 marks)

7. A car of mass 900 kg has a power output of 6000 W, and moves against a resistance kv, where v is its speed, and k is a constant. When it is moving on a level road, its maximum speed is 40 ms^{-1}.

   (a) Find the value of the constant k. (5)

   (b) Find the acceleration of the car when it is travelling at 5 ms^{-1} up an incline whose angle to the horizontal is θ, where $\sin\theta = 1/30$. (5)

(Total: 10 marks)

8. An aircraft can fly at 200 km h^{-1} in still air. The pilot wishes to set a course so that he can fly from town A to town B, which is 500 km from A on a bearing of 030^0. There is wind blowing from the south-east at a speed of 40 km h^{-1}.

   (a) What course should the pilot set? (7)

   (b) How long will it take to travel from A to B? (3)

(Total: 10 marks)

9. A sailing ship is rolling in heavy seas. A sailor is at the top of a tall mast which swings from side to side. While the sailor is in contact with the mast, his motion is modelled as that of a particle moving horizontally with simple harmonic motion of period 10 s and amplitude 7 m. The mass of the sailor is 100 kg and he loses his grip when the horizontal force acting on him is 245 N.

   (a) Find his distance from the centre of the oscillation at the instant when he loses his grip. (5)

   (b) Find his speed when he loses his grip. (5)

(Total: 10 marks)

10. A ball of mass 2 kg is attached to one end of an elastic string. The other end is fixed to the ceiling. The string has a natural length of 2 m and a modulus of elasticity of 100 N.

The ball is held so that the string is at its natural length and is then released from rest. In the subsequent motion, the velocity of the ball is v when the ball is at a depth x below its initial position. It can be assumed that air resistance can be ignored.

(a) Using conservation of energy, obtain a relation between v and x. (5)

(b) Find the velocity of the ball when $x = 0.2$ m. (1)

(c) Find the distance through which the ball drops before it comes to instantaneous rest. (2)

(d) Find the maximum velocity of the ball during the motion. (2)

(Total: 10 marks)

AM 02/II.17s

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MATRICULATION EXAMINATION
ADVANCED LEVEL

SEPTEMBER 2017

SUBJECT:	**APPLIED MATHEMATICS**
PAPER NUMBER:	II
DATE:	**5th September 2017**
TIME:	**9.00 a.m. to 12.05 p.m.**

Directions to candidates

Answer **SEVEN** questions.

In all there are 10 questions each carrying 15 marks.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

In this paper, **i**, **j**, **k** are unit vectors along the x-, y- and z-axes of a Cartesian coordinate system.

(Take $g = 10$ ms^{-2}).

1. A rigid uniform beam AB has weight $2W$ and length $2l$. The beam's weight is distributed uniformly throughout its length. The beam is supported horizontally at its end-points A and B, and carries a concentrated load $6W$ at its midpoint. This structural system is in equilibrium.

   (a) Find the normal reactions at the supports of the beam. (2)

   (b) Find the shearing force and bending moment at any point along the beam. (9)

   (c) Draw a sketch of the shearing force and bending moment, and deduce where they are largest. (4)

(Total: 15 marks)

2. A force system consists of three forces as follows:

   $\mathbf{F}_1 = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ acting at the point with position vector $3\mathbf{i} - \mathbf{j} + \mathbf{k}$,

   $\mathbf{F}_2 = \mathbf{i} + 3\mathbf{j} - \mathbf{k}$ acting at the point with position vector $\mathbf{j} + 2\mathbf{k}$,

   $\mathbf{F}_3 = s\mathbf{i} + t\mathbf{j} + 2\mathbf{k}$ acting at the point with position vector $\mathbf{k}$.

   (a) Obtain, in terms of s and t, the equivalent force system consisting of a single force $\mathbf{F}$ acting through the origin and a couple of moment $\mathbf{G}$. (11)

   (b) If $\mathbf{G}$ is parallel to $\mathbf{F}$, find the values of s and t. (4)

(Total: 15 marks)

3. At time $t = 0$ s, particles A and B are at points with position vectors $2\mathbf{i} + \mathbf{j}$ m and $3\mathbf{i} + 4\mathbf{j}$ m respectively. They have constant velocities of $-\mathbf{i} + 2\mathbf{j}$ ms^{-1} and $p\mathbf{i} + q\mathbf{j}$ ms^{-1} respectively.

   (a) Show that, if the particles are to collide at some time after the start, then $p < -1$, $q < 2$ and $q = 3p + 5$. (6)

   (b) Show that, if the particles are closest to each other at some time after the start, then $p + 3q < 5$. (5)

   (c) If $p = -2$ and $q = 2$, find the time when the particles are closest to each other, and find the distance which separates them at that instant. (4)

(Total: 15 marks)

4. Two smooth spheres, A and B, of equal radii collide. The mass of A is 3 kg and that of B is 2 kg.

   Sphere A has a velocity of $2\mathbf{i}+3\mathbf{j}$ ms^{-1}, whilst B has a velocity of $-3\mathbf{i}-4\mathbf{j}$ ms^{-1}, where the **i**-direction is along the line of centres from A to B at the moment of impact. The coefficient of restitution between the two spheres is 0.8.

   Find:

   (a) the velocities of the spheres after impact; (9)

   (b) the kinetic energy lost during the collision; (3)

   (c) the impulse which each sphere exerts on the other during the collision. (3)

(Total: 15 marks)

5. A hollow cylinder, of internal radius 2 m, rests with its axis horizontal. O is a point on that axis. A particle, P, of mass 5 kg, rests inside the cylinder vertically below O. It is given an impulse so that it starts to move on the smooth surface of the cylinder in a circle about O. Its initial speed is 8 ms^{-1}.

   (a) Find the normal reaction between the cylinder and the particle when the line OP makes an angle θ with the downward vertical? (10)

   (b) What angle does OP make with the downward vertical when the normal reaction is zero? (3)

   (c) What is the speed of the particle when the normal reaction is zero? (2)

(Total: 15 marks)

6. A particle of mass 1 kg falls vertically from rest. A resisting force equal to $2v$ N acts on the particle during its descent, where v ms^{-1} is the speed of the particle at time t seconds after it begins to fall.

   (a) Using Newton's Second Law of Motion, set up a differential equation relating the speed v to the time t. (2)

   (b) By integrating this differential equation, show that the velocity is given by the equation $v(t) = \frac{1}{2}g(1 - e^{-2t})$. (6)

   (c) Draw a sketch of the graph of $v(t)$ against t, showing clearly the behaviour of v as t becomes large. (3)

   (d) By integrating v with respect to t, obtain an expression for the displacement $x(t)$ of the particle as a function of t. (4)

(Total: 15 marks)

7. Two particles, each of mass m, are connected by a light inextensible string and are placed on the rough outer surface of a sphere, with one of the particles at the highest point of the sphere. The string is taut and the second particle is as low as possible on the surface of the sphere. The string subtends an angle θ at the centre of the sphere, with $\theta < \pi/2$. The coefficient of friction between the sphere and each particle is μ.

   Assuming friction is limiting for both particles, show that

$$\mu = \frac{\sin\theta}{1 + \cos\theta}.$$

(Total: 15 marks)

8. A uniform rod AB has mass m and length $2a$. The rod can rotate in a vertical plane about a fixed smooth horizontal axis passing through A and perpendicular to the plane of rotation.

   (a) Find by integration the moment of inertia of the rod about this axis. (5)

   (b) The rod is gently displaced from rest with B vertically above A. Find the angular velocity of the rod when B reaches a point vertically below A. (5)

   (c) If, when B is vertically below A, the angular velocity is in fact half that found in (ii), find the magnitude of the constant frictional couple at the axis. (5)

(Total: 15 marks)

9. A uniform circular disc has mass m, radius a and centre O. P is a point on the circumference of the disc.

   (a) Find by integration the moment of inertia of the disc about an axis passing through O and perpendicular to the disc. Deduce by the theorem of parallel axis, the moment of inertia about an axis passing through P and perpendicular to the disc. (5)

   (b) The disc is spinning freely about its stationary centre with angular velocity ω, with its plane horizontal and resting on a smooth horizontal surface. The point P on the circumference is suddenly fixed. Using conservation of momentum, or otherwise, find the new angular velocity of the disc. (7)

   (c) Find the ratio of the kinetic energy of the system before and after the occurrence. (3)

(Total: 15 marks)

10. A uniform, circular lamina, has centre O, mass m and radius a. It is free to rotate about a fixed smooth horizontal axis through O perpendicular to the lamina. A particle of mass m is attached to the lamina at a point P on its rim. The lamina is released from rest with OP horizontal.

    (a) Find the angular velocity and angular acceleration of the rod in terms of the angular displacement. (6)

    (b) Find the force on the axis when the lamina has rotated through 60°. (9)

(Total: 15 marks)