MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION CERTIFICATE EXAMINATION
INTERMEDIATE LEVEL

MAY 2012

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**15th May 2012**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. A motorist starts a car from rest, and accelerates uniformly to a speed of v ms^{-1} in 9 seconds. He maintains this speed for another 50 seconds, and then applies the brakes and decelerates uniformly to rest. His deceleration is numerically equal to three times his previous acceleration.

   (i) Sketch a velocity-time graph for this motion.

   (ii) Calculate the time during which the deceleration takes place.

   (iii) Given that the total distance travelled is 840 m, calculate the value of v.

   (iv) Calculate the initial acceleration.

   [4, 2, 3, 1 marks]

2. A set of Cartesian axes has origin O. Unit vectors acting along the positive x-axis and y-axis are denoted by $\mathbf{i}$ and $\mathbf{j}$ respectively.

   A square OABC lies in the x-y plane, with its vertex O at the origin, with the vertex A at $4\mathbf{i} + 3\mathbf{j}$, and the vertex C at $3\mathbf{i} - 4\mathbf{j}$. Forces of magnitude 5 N, $10\sqrt{2}$ N and 10 N act along OA, OB and CO respectively, in the directions indicated by the order of the letters.

   (i) Using the parallelogram law of addition, or otherwise, find the coordinates of the vertex B of the square.

   (ii) Find the resultant of the forces in $\mathbf{i}$, $\mathbf{j}$ notation.

   (iii) Find the Cartesian equation of the line of action in which the resultant acts.

   (iv) Find the moment of the resultant about C.

   [2, 4, 2, 2 marks]

3. A can in the form of a circular cylinder, without a lid, is made of thin metal sheeting of uniform thickness and with a mass per unit area of 1 g cm^{-2}. The radius of the can is 10 cm, and its height is 20 cm. The can is placed with its base on a horizontal plane, and is half-filled with a liquid of density 1.5 g cm^{-3}.

   (i) Find the height of the centroid of this system from the base of the can.

   (ii) When a particle of mass m grams is attached to the centre of the base of the can, the centroid of the system is at a height of 5 cm from the base of the can. Find the value of m in grams.

   [7, 3 marks]

4. A sphere of mass m moving along a smooth horizontal table with speed V collides directly with a stationary sphere of the same radius and of mass $2m$. The coefficient of restitution for this impact is e.

   (i) Find the velocities of the two spheres after impact in terms of V and e.

   (ii) If half the kinetic energy is lost on impact, find the value of e.

   [6, 4 marks]

5. A ball is thrown from a balcony above a horizontal lawn. The velocity of projection is 10 ms^{-1} at an angle of elevation α above the horizontal, where $\tan\alpha = \frac{3}{4}$. The ball moves freely under gravity, and hits the lawn at a point A after 3 s. Calculate:

   (i) the horizontal distance between the point of projection and the point A;

   (ii) the vertical height above the lawn from which the ball was thrown;

   (iii) the angle, to the nearest degree, between the direction of the velocity of the ball at A and the horizontal.

[2, 5, 3 marks]

6. A block of weight 1.5 kg lying on a rough inclined plane is prevented from slipping down the plane by a string attached at one end to a point of the block and, at the other end, to a fixed point. The string is parallel to a line of greatest slope of the plane. The angle of inclination of the plane to the horizontal is α where $\sin\alpha = \frac{4}{5}$, and the coefficient of friction between the block and the plane is 0.5.

   (i) Calculate the tension of the string in newtons.

   (ii) If the string is cut, find the acceleration with which the block slides down the plane.

[7, 3 marks]

7. A right circular solid cone of semi-vertical angle 45° has its base fixed to horizontal ground, with its vertex above the level of the ground. A light inextensible string, which has one end attached to the vertex of the cone, is 1 m long, and carries at its other end a particle of mass 1 kg, which rests on the exterior surface of the cone. The particle can rotate in a horizontal circle in contact with the *smooth* outer surface of the cone.

   (i) Draw a diagram showing the forces on the particle.

   (ii) Assuming the angular velocity of the particle is constant and equal to ω, and that the particle remains in contact with the cone, obtain expressions in terms of ω for the tension in the string and for the cone's reaction on the particle.

   (iii) Find the maximum value of ω if the particle is to remain in contact with the cone.

[2; 3, 3; 2 marks]

8. A water skier of mass 95 kg is towed by a horizontal rope behind a boat. His body is straight, and the thrust of the water acts along the line of his body.

   (i) When moving with uniform velocity, he is leaning back at 10^o to the vertical. Find the tension in the rope.

   (ii) The boat starts to accelerate, and the skier leans back at 15^o to the vertical. If the tension in the rope is now 500 N, find the acceleration of the boat.

[4, 6 marks]

9. A girl on a sledge slides down a hill of variable gradient. In so doing, she travels a distance of 168 m, measured along the surface of the track, and descends a vertical distance of 30 m. The combined mass of the girl and the sledge is 80 kg. If the initial speed is 2 ms^{-1}, and the final speed is 16 ms^{-1}, find:

   (i) the increase in kinetic energy of the combined mass of girl and sledge;

   (ii) the work done by gravity;

   (iii) the average resistance to the motion.

[3, 3, 4 marks]

10. Two light rings can slide on a rough horizontal rod. The rings are connected to the ends A and B of a light inextensible string AB. A particle of weight W is then attached to the midpoint C of the string. The coefficient of friction between either ring and the rod is μ. The system is in limiting equilbrium when the angle CAB is equal to 60^o.

   (i) Draw a diagram showing clearly the forces on the rings and on the particle.

   (ii) Find the coefficient of friction, μ.

   (iii) Find the tension in the string.

[4, 3, 3 marks]