MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION CERTIFICATE EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2012

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**10th September 2012**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ $\mathbf{ms^{-2}}$).

1. Forces of magnitude 2 N, 4 N and 4 N act along the sides AB, BC, CA respectively of an equilateral triangle ABC of side 2 m, the forces being in the direction defined by the order of the letters.

   (i) Find the magnitude and direction of their resultant, and the point where its line of action cuts AC produced.

   (ii) This system of forces is to be reduced to equilibrium by the addition of a couple in the plane ABC and a force which acts through A. Find the magnitude and direction of the force, and the magnitude and sense of the couple.

   [5, 5 marks]

2. The brakes of a train, which is travelling at 30 ms^{-1}, are applied as the train passes point A. The brakes produce a constant retardation of magnitude $3f$ ms^{-2} until the speed of the train is reduced to 10 ms^{-1}. The train travels at this speed for a distance and is then uniformly accelerated at f ms^{-2} until it again reaches a speed of 30 ms^{-1} as it passes point B. The time taken by the train in travelling from A to B, a distance of 4 km, is 4 minutes.

   (i) Draw a speed-time graph for this motion.

   (ii) Find the value of f.

   (iii) Find the distance travelled at 10 ms^{-1}.

[3, 6, 1 marks]

3. A uniform wire ABC of mass 12 grams is bent into a triangle ABC, right-angled at A, and with AB = 4 cm, BC = 5 cm and CA = 3 cm. Two masses of 5 grams and m grams are attached to B and C respectively.

   (i) Find, in terms of m, the distances of the centroid G of the system from AB and BC.

   (ii) The triangle hangs freely from A. If the system is in equilibrium with BC horizontal, find the value of m.

[5, 5 marks]

4. Three beads A, B and C, of masses $3m$, $2m$ and m respectively, are threaded in that order on a smooth horizontal wire. Initially, A, B and C are separated and at rest. Then, A is projected towards B with speed $10V$. The coefficient of restitution between A and B is 0.5.

   (i) Show that the speed of B after the collision is $9V$.

   (ii) Find the velocity of A after the collision.

   (iii) Find the kinetic energy lost in the collision.

   (iv) After A and B have collided, C is projected towards B with speed $6V$. If the collision between B and C is perfectly elastic ($e = 1$), find the velocities of B and C after this collision.

[3, 2, 2, 3 marks]

5. A ball is projected upwards from a point on a horizontal plane, with speed 40 ms^{-1}, at an angle of 60^o to the horizontal. The point of projection is at a horizontal distance of 40 m from the foot of a vertical wall which is 10 m high. It can be assumed that there is no resistance to the motion, and that the motion takes place in a plane perpendicular to the wall. Find:

   (i) the vertical height by which the ball clears the wall;

   (ii) the greatest height above the horizontal plane reached by the ball;

   (iii) the time of flight;

   (iv) the horizontal distance beyond the wall at which the ball strikes the plane.

[4, 2, 2, 2 marks]

6. A non-uniform rod AB of length a and weight W has its centroid at the point G, where $\text{AG} = 3a/4$. The rod is smoothly pivoted at A, and is held in equilibrium by a light string BC, also of length a, joining the end B of the rod to a point C vertically above A. The triangle ABC is in a vertical plane, with AC vertical, and with angles BAC and ACB both equal to 30^o. Find:

   (i) the tension in the string, and

   (ii) the magnitude of the reaction at the hinge.

[5, 5 marks]

7. A block of mass 1.5 kg lying on a rough inclined plane is prevented from slipping down the plane by a string attached at one end to a point of the block and, at the other end to a fixed point. The string is parallel to a line of greatest slope of the plane. The angle of inclination of the plane to the horizontal is α, where $\sin\alpha = 4/5$, and the coefficient of friction between the block and the plane is 0.5.

   (i) Find the tension in the string.

   (ii) If the string is cut, find the acceleration with which the block slides down the plane.

[5, 5 marks]

8. A light framework ABCD consists of four light rods AB, BC, CD and DA, arranged in the form of a trapezium. In this trapezium, the sides have lengths AB = $2a$, and BC = CD = DA = a, the sides AB and CD are parallel, and the interior angles at A and B are both equal to 60°. All rods are smoothly jointed at A, B, C and D.

   The framework is freely suspended at A and B, with AB horizontal, and with CD below AB. Two weights, each equal to W, are then attached to joints C and D respectively. The system is in equilibrium.

   (i) Draw a diagram of the system, showing all the forces acting in it.

   (ii) Find the reactions at the supports A and B.

   (iii) Find the forces in the rods, indicating whether they are in tension or in compression.

[3, 1, 6 marks]

9. A bend in a road is in the form of a circular arc of radius r, and is banked at 20° to the horizontal. A lorry, travelling around the bend at 20 ms^{-1}, is on the verge of slipping down this slope. The coefficient of friction between the lorry's tyres and the road is 0.2.

   (i) Draw a diagram showing all the forces acting on the lorry.

   (ii) Find the value of r.

[3, 7 marks]

10. A ball of mass 2 kg is attached to one end of an elastic string, whose other end is fixed to the ceiling. The string has a natural length of 2 m and a modulus of elasticity of 100 N. The ball is held so that the string is at its natural length, and is then released from rest. When the ball has descended x metres, its velocity is v ms^{-1}.

    (i) Using the principle of conservation of energy, obtain a relation between v and x.

    (ii) Find the speed when the ball has descended by 0.4 m.

    (iii) Find the distance the ball drops before coming to instantaneous rest.

[7, 1, 2 marks]