IM 02.13s

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2013

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**7th September 2013**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ ms^{-2}).

1. ABCD is a rectangle with side AB = 4 m and side AD = 3 m. Forces of magnitude 10 N, 15 N and 5 N act along $\overrightarrow{AC}$, $\overrightarrow{DB}$ and $\overrightarrow{BC}$ respectively.

   (i) Find the magnitude and direction of the resultant, and where the line of action of the resultant cuts BA, produced if necessary.

   (ii) The system of forces is equivalent to a force acting at the centre of the rectangle together with a couple. Find the magnitude and direction of this force and the magnitude and sense of the couple.

[7, 3 marks]

2. A uniform hexagonal lamina ABCDEF has weight W. The lamina is smoothly pivoted at A. It is kept in equilibrium in a vertical plane with AB horizontal, and with the vertices C, D, E and F above AB, by a force of magnitude P acting at E in the direction of $\overrightarrow{BE}$.

   (i) Find the force P and the magnitude and direction of the force exerted on the lamina by the pivot.

   (ii) The force at E is reversed in direction while retaining its magnitude. Find the weight which must be attached at F to keep the lamina in this position of equilibrium.

[7, 3 marks]

3. A solid consists of a uniform square lamina ABCD of side $2a$, to which is glued a uniform circular lamina of radius a, and of the same material as the square. The side BC of the square coincides with a diameter of the circular lamina.

   (i) Find the distance of the centre of gravity of the composite solid from the side AD of the square.

   (ii) The solid is then suspended freely from the midpoint of CD, and is held in equilibrium with AB horizontal by means of a weight of mass M_1 attached to A. If M_0 is the mass of the solid on its own, find the ratio M_1/M_0.

[6, 4 marks]

4. A particle A, moving in a straight line with constant acceleration f passes a point O with velocity 12 ms^{-1}. After 1 s, another particle B, moving along the same straight line with constant acceleration $7f/4$ passes O with velocity 9 ms^{-1}. B overtakes A after another 4 s.

   (i) Find the numerical value of f.

   (ii) Find the velocities of A and B at the instant when B overtakes A.

   (iii) If B stops accelerating on overtaking A and continues moving with constant velocity, find the time A takes to overtake B.

[5, 2, 3 marks]

5. The position of a tennis ball, t seconds after it is hit, is given by:

$$\mathbf{r}(t) = 10t\mathbf{i} + (1 + 20t - 5t^2)\mathbf{j},$$

where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical constant unit vectors respectively, and distance is measured in metres. Taking the origin at ground level, find:

(i) the velocity and acceleration of the ball at time t;

(ii) the greatest height reached by the ball and the horizontal distance travelled up to that point;

(iii) the time when the ball is moving in the direction of the vector $2\mathbf{i} + \mathbf{j}$;

(iv) the Cartesian equation of the path.

[2, 3, 3, 2 marks]

6. A gun of mass 800 kg fires a shell of mass 4 kg horizontally at 400 ms^{-1}. The gun rests on rough horizontal ground, and the coefficient of friction between the gun and the ground is 0.6. The gun is stationary before the shot is fired. Find:

(i) the velocity of the gun immediately after it is fired;

(ii) the impulse on the gun generated by the shell;

(iii) the distance travelled by the gun before coming to rest.

[2, 1, 7 marks]

7. The resistance to motion of a car is proportional to the square of its speed. The car has a mass of 1000 kg and can maintain a steady speed of 30 ms^{-1} when travelling up a hill inclined at $\sin^{-1}(1/20)$ to the horizontal with the engine working at 60 kW.

(i) Obtain an expression for the resistance to motion in terms of the speed of the car.

(ii) The car then travels down the same hill with the engine working at 40 kW. Find the acceleration of the car at the instant when the speed is 20 ms^{-1}.

[5, 5 marks]

8. Four particles, each of mass 0.03 kg, are connected by light inextensible strings, each of length 0.08 m, so that they form a square with the particles at the corners and the strings forming the sides. The particles are placed in this configuration symmetrically on a turntable, which is made to rotate with angular speed 2 rad s^{-1}. It can be assumed that the masses are stationary with respect to the turntable, and that friction can be ignored.

   (i) Find the tension in the strings.

   (ii) If the strings break when the tension in them exceeds 0.05 N, find the maximum possible angular speed of the turntable if the strings are not to break.

[7, 3 marks]

9. An elastic string has one end fixed to a point A. The other end B, which is attached to a particle of mass 2 kg, is pulled vertically down from A until AB is 3 m and then released. The modulus of elasticity of the string is 22 N and its natural length is 1 m. Find:

   (i) the velocity of the particle at the instant when AB is 2 m;

   (ii) the velocity of the particle at the instant when AB is 1 m;

   (iii) the highest position, relative to A, reached by the particle.

[5, 3, 2 marks]

10. A uniform rod AB, of weight W and length $2a$, rests in equilibrium in a vertical plane with one end A on rough horizontal ground and the point C of the rod in contact with a smooth fixed peg. The length AC is $3a/2$, and the rod makes an angle of 30° with the vertical. Find:

    (i) the reaction exerted by the peg on the rod;

    (ii) the normal reaction and the frictional force exerted by the ground on the rod at A;

    (iii) the minimum value of the coefficient of friction for equilibrium to be possible.

[4, 4, 2 marks]