IM 02.14s

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2014

SUBJECT:	**APPLIED MATHEMATICS**
DATE:	**6th September 2014**
TIME:	**9.00 a.m. to 12.00 noon**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

(Take $g = 10$ $\mathbf{ms}^{-2}$).

1. Forces F, $2F$, $3F$ and $4F$ acting along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ of a square ABCD of side $2a$ are to be replaced by an equivalent system made up of three forces acting along the sides of the triangle ABC.

   (i) Find, in terms of F, the magnitude of each of these forces.

   (ii) If the force acting along AC is now reversed, find the distance from A of the point where the line of action of the resultant now cuts AB, produced if necessary.

[6, 4 marks]

© The MATSEC Examinations Board reserves all rights on the examination questions in all papers set by the said Board.

2. The brakes of a train, which is travelling at 30 ms^{-1}, are applied as the train passes point A. The brakes produce a constant retardation of magnitude 3λ ms^{-2} until the speed of the train is reduced to 10 ms^{-1}. The train travels at this speed for a distance, and is then accelerated uniformly at λ ms^{-2} until it again reaches a speed of 30 ms^{-1} as it passes point B. The time taken by the train in travelling from A to B, a distance of 4 km, is 4 minutes.

   (i) Sketch the speed-time graph for this motion.

   (ii) Find the value of λ.

   (iii) Find the distance travelled at 10 ms^{-1}.

[3, 6, 1 marks]

3. Three uniform rods AB, BC and CA are arranged in the form of a triangular framework ABC, which is right-angled at A. AB is of length 4 m, whilst AC is of length 3 m. The rods have a mass of 10 kg/m. Particles, each of mass 20 kg, are placed at the vertices A, B, C of the framework.

   (i) Find the distance of the centre of mass of this system from the sides AC and AB.

   (ii) The solid is then suspended freely from the vertex C. Find the angle which AC makes with the downward vertical.

[8, 2 marks]

4. Two small spheres P and Q move directly towards each other on a smooth horizontal table with speeds $4u$ and $2u$ respectively. The mass of P is $2m$, the mass of Q is m, and the coefficient of restitution between the spheres is e. After colliding, P and Q both move in the same direction with speeds v and w respectively.

   (i) Find v and w in terms of e and u.

   (ii) The total kinetic energy of the spheres after the collision is k times their total kinetic energy before the collision. Show that $k = (2e^2 + 1)/3$.

   (iii) Comment on the case when $k = 1$.

[6, 3, 1 marks]

5. A stone is thrown with a horizontal velocity of 10 ms^{-1} from the top of a tower of height h m. At the same instant, a stone is thrown from the foot of the tower with speed 20 ms^{-1} at an angle of 60° above the horizontal. Find:

   (i) the height of the tower if the stones collide after $\sqrt{3}$ s, and

   (ii) the distance of the stones from the foot of the tower when they collide.

[8, 2 marks]

6. A car of mass 1000 kg moves with its engine shut off down a slope of inclination α, where $\sin\alpha = 1/20$, at a steady speed of 15 ms^{-1}.

   (i) Find the resistance, in newtons, to the motion of the car.

   (ii) Calculate the power delivered by the engine when the car ascends the same inclination at the same steady speed, assuming that the resistance to motion is unchanged.

[4, 6 marks]

7. A particle P of mass 1 kg is attached to one end of a light inextensible string of length 0.75 m. The other end of the string is fixed to point O on a smooth horizontal table. The particle moves in a horizontal circle of radius 0.75 m with constant angular speed ω rad/s.

   (i) Find the value of ω and the speed of the particle if the tension in the string is 24 N.

   (ii) A particle of mass 2 kg is placed at rest directly in the path of P. The particles stick together on impact. Find the velocity of the composite particle after the impact.

   (iii) Calculate the tension in the string in the subsequent circular motion.

[6, 2, 2 marks]

8. A uniform ladder rests with one end against a rough wall and the other end on rough horizontal ground. The coefficient of friction between the ladder and the wall, and the ladder and the ground is μ. The ladder is on the point of slipping when it is inclined at 30^o to the vertical. It can be assumed that friction is limiting at both the wall and the ground.

   Find the value of μ.

   [10 marks]

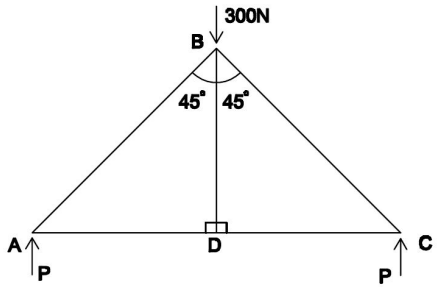
9. A light elastic string of natural length 0.75 m has one end attached to a fixed point O, and carries at the other end a mass of 0.80 kg. When the mass hangs in equilibrium, the string stretches by 0.20 m.

   (i) Find the modulus of elasticity of the string.

   (ii) The mass is lifted so that it is at O, and then released from rest. Subsequently, the particle first comes to rest at a point Q, vertically below O. Find the distance OQ.

   [3, 7 marks]

10. A light framework ABCD is made up of five uniform rods AB, BC, CD, DA and DB smoothly jointed together as in the figure below.



   The framework lies in a vertical plane, and rests with ADC horizontal on two supports at A and C. The framework carries a load of 300 N at B. Find:

   (i) the reaction P at the supports;

   (ii) the forces in the rods, stating whether they are in tension or in compression.

   [2, 8 marks]