MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2013

SUBJECT:	PURE MATHEMATICS
DATE:	9th May 2013
TIME:	9.00 a.m. to 12.00 noon

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

---

1. (a) Find the value of x if

$$\frac{3^{x+4}}{27^x} = \frac{9^{2x-1}}{3^{x+1}}.$$

(b) In an ant colony, the number of ants is given by $n(t) = n_0 e^{\lambda t}$, where t is in days, and n_0 and λ are constants. Initially, the number of ants is 100, and the number of ants doubles after 5 days. Write down the constant n_0, and estimate λ to four places of decimals.

(c) Find the values of A, B and C in the expression

$$f(x) = \frac{9}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}.$$

Hence find $\int f(x)dx$.

[3; 3; 4, 3 marks]

2. (a) The function $f(x)$ is given by $f(x) = x^3 + ax^2 - 7x + b$. The factor $(x-3)$ divides $f(x)$ exactly, and the remainder when $f(x)$ is divided by $(x-1)$ is -12.

   (i) Using the remainder theorem, or otherwise, find the values of a and b.

   (ii) Given that the two other roots of $f(x)$ are negative integers, factorise the polynomial $f(x)$.

   (iii) Draw a sketch of $f(x)$, and find where $f(x) \leq 0$.

   (b) The quadratic equation $x^2 + x - 2 = 0$ has roots α and β. Find the quadratic equation whose roots are $\dfrac{\alpha}{\beta}$ and $\dfrac{\beta}{\alpha}$.

[4, 1, 3; 4 marks]

3. (a) In an arithmetic progression, the sum of the first four terms is 8, whilst the sum of the 6th and 7th terms is -12. Find the initial term, the common difference, and the sum of the first 100 terms.

   (b) In a geometric progression, the sum of the first 3 terms is 10, whilst the sum to infinity is 80/7. Find the initial term, the common ratio and the sum of the first ten terms.

[4, 4 marks]

4. (a) Find the equation of the perpendicular bisector of the line joining the points A(1, 2) and B(5, 6).

   (b) A function $f(x)$ is defined by $f(x) = \sin 2x$.

   (i) Draw a sketch of $f(x)$ for $0 \leq x \leq 2\pi$.

   (ii) Find the range of $f(x)$.

   (iii) If $g(x) = f(x - \frac{\pi}{2})$, draw a sketch of $g(x)$ on the same sketch as in (i) above.

   (iv) Find, within the interval $0 \leq x \leq \frac{\pi}{2}$, the values of x for which $f(x) \geq \dfrac{1}{\sqrt{2}}$.

[4; 3, 1, 2, 3 marks]

5. (a) Using the binomial expansion, expand $(3 - 5x)^{10}$ in ascending powers of x up to and including the term in x^2. Using an appropriate value of x in this expansion, obtain to the nearest integer an approximate estimate of 2.995^{10}.

(b) The probability that student A wears a white top to University on a given day is 0.2, whilst the probability that student B wears a white top is 0.15. The probability that both A and B wear a white top on the same day is 0.09.

(i) Using a Venn diagram, or otherwise, find the probability that either or both A and B wear a white top.

(ii) Find the probability that exactly one of A or B wears a white top on a given day.

(c) Tom rolls a pair of fair dice. Find the probability that he gets a total score of 7.

[4; 2, 1; 3 marks]

6. (a) If C is given by the equation $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}C = -2\sqrt{3}+5$, find the value of C in the form $a\sqrt{3}+b$, where a and b are integers.

(b) A regular polygon of n sides is inscribed in a circle of unit radius.

(i) Find the area of this polygon in terms of n.

(ii) Find, to the nearest integer multiple of 10, the value of n above which the difference in areas between the circle and the polygon is less than 2% of the area of the circle.

[3; 4, 3 marks]

7. (a) Differentiate the following functions with respect to x:

$$f(x) = \frac{\sin x}{1+x}, \qquad g(x) = x^3 e^{-2x}, \qquad h(x) = \ln(3 + \sin 2x).$$

(b) Using differentiation, find the coordinates of the two turning points of the function $f(x) = x^3 - 12x + 1$.

[1, 2, 2; 4 marks]

8. (a) An initial value problem is defined by the differential equation

$$x\frac{dx}{dt} = t + 1.$$

It can be assumed that $x = 2$ when $t = 1$, and that $x(t)$ is positive if $t \geq 0$.

Find $x(t)$ using separation of variables.

(b) Using integration, find the area enclosed by the quadratic $y = x^2 - 5x + 4$ and the line $y = 1 - x$.

[4; 5 marks]

9. A 2×2 matrix **A** represents a reflection in the line $y = x$, whilst a 2×2 matrix **B** represents an anti-clockwise rotation through $\pi/2$.

(i) Write down the matrices **A** and **B**.

(ii) Find the matrix products **BA** and $(\mathbf{BA})^2$.

(iii) Interpret geometrically the results obtained in (ii) above.

[2, 2, 2 marks]

10. (a) A matrix **A** is given by $\mathbf{A} = \begin{pmatrix} 1 & -2 \\ a & 3 \end{pmatrix}$, where a is a constant.

(i) Find the determinant of **A**, and deduce the values of a for which the inverse matrix, $\mathbf{A}^{-1}$, exists.

(ii) Find $\mathbf{A}^{-1}$ in terms of a.

(b) The matrices **B** and **C** are given by $\mathbf{B} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$, and $\mathbf{C} = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$.

(i) Find the matrix product **BC**.

(ii) Deduce $\mathbf{B}^{-1}$.

(iii) Using the method of the matrix inverse, solve the system of linear equations:

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}.$$

[2, 2; 2, 1, 3 marks]