MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

MAY 2014

SUBJECT:	PURE MATHEMATICS
DATE:	13th May 2014
TIME:	9.00 a.m. to 12.00 noon

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

---

1. (a) Find the value of x and y if

$$xe^y = 10,$$
$$x^2e^{3y} = 20.$$

(b) Find the positive value of x if

$$\log(x^2 + 2x + 1) - \log(x + 1) = \log(2x - 1) + \log(3x - 1).$$

(c) Using surds, express the fraction $\dfrac{3\sqrt{2}+5}{2\sqrt{2}-1}$ in the form $(a + b\sqrt{2})/c$ where a, b and c are integers.

[4; 4; 3 marks]

2. (a) Find the acute angle x such that $5\cos x - 2\sin^2 x - 1 = 0$.

   (b) A function $f(x)$ is defined as:

$$f(x) \quad = \quad \frac{3}{(2x+1)(x+2)} \quad \equiv \quad \frac{A}{2x+1} + \frac{B}{x+2}.$$

   (i) Find by partial fractions the values of A and B in this representation of $f(x)$.

   (ii) Find $\int f(x)dx$.

[4; 4, 3 marks]

3. (a) An athlete is training for the annual race organised by her school. On the first day she runs 500 metres. Subsequently, she increases her running distance by 100 metres every day for 29 more days.

   Find the total distance run by the athlete over these 30 days.

   (b) A geometric series is defined by:

$$2 + \frac{3}{2} + \frac{9}{8} + ...$$

   (i) Find the sum to infinity of this series.

   (ii) How many terms should be taken from this series in order that their sum exceeds 7.99?

[4; 2, 5 marks]

4. The quadratic function $p(x)$ is given by $p(x) = x^2 + x$.

   (i) Using differentiation, find the minimum value of $p(x)$.

   (ii) Write down the range of $p(x)$.

   (iii) Draw a sketch of $p(x)$ showing clearly the minimum and the points where it cuts the coordinate axes.

   (iv) Show that the equation of the tangent to the graph of $p(x)$ at $x = a$ is given by

$$y = (2a+1)x - a^2.$$

   (v) Another quadratic $q(x)$ is defined as $q(x) = x^2 + x + C$, where C is a constant. The quadratic $q(x)$ leaves a remainder of 5 when it is divided by $(x-2)$. Using the remainder theorem, or otherwise, find the value of C.

[3, 1, 2, 3, 3 marks]

5. (a) A function $f(x)$ is defined by $f(x) = (1+2x)^{100}$.

   (i) Using the binomial expansion, expand $f(x)$ in ascending powers of x up to and including the term in x^2.

   (ii) Using this expansion, obtain an approximation to 1.002^{100}.

   (b) The word *LOCATED* has 7 letters in all, 4 consonants and 3 vowels.

   (i) Find the number of different permutations of the seven letters in this word.

   (ii) In how many of these permutations do the three vowels occur at the end?

   (iii) Find the probability that a permutation has its three vowels occurring at the end.

[4, 2; 2, 2, 1 marks]

6. A regular pentagon ABCDE has side 2 cm.

   (i) Find the internal angle of this pentagon.

   (ii) Show that AD is parallel to BC.

   (iii) Find the area of the pentagon to two places of decimals.

   (iv) A circle is drawn passing through the vertices of the pentagon. Find the area of a segment bounded by this circle and one of the sides of the pentagon.

[1, 3, 3, 3 marks]

7. (a) Differentiate the following functions with respect to x:

$$f(x) = xe^{-x}, \qquad g(x) = (\sin x + 1)^5.$$

   (b) A thin wire of length 2 cm is bent into the shape of an isosceles triangle with one side of length $2x$ cm and two equal sides.

   (i) Find the area $A(x)$ of this triangle in terms of x. Hence find $[A(x)]^2$.

   (ii) By differentiating $[A(x)]^2$ with respect to x, or otherwise, find the value of x for which the area is maximum.

[1, 1; 4, 2 marks]

8. (a) An initial value problem is defined by the differential equation

$$\frac{dx}{dt} = \frac{t+1}{t^2+2t+3}.$$

It can be assumed that $x = 0$ when $t = 0$. Find $x(t)$ using separation of variables.

(b) Find $\int_0^{2\pi} 1 + \sin t \, dt$. Draw a sketch of this integral.

[4; 5 marks]

9. A 2×2 matrix $\mathbf{A}$ represents a reflection in the x-axis, whilst a 2×2 matrix $\mathbf{B}$ represents a reflection in the y-axis.

(i) Write down the matrices $\mathbf{A}$ and $\mathbf{B}$.

(ii) Find the matrix products $\mathbf{BA}$ and $(\mathbf{BA})^2$.

(iii) Interpret geometrically the results obtained in (ii) above.

[2, 2, 2 marks]

10. (a) Three matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{A} = \begin{pmatrix} a & b \\ 1 & 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 11 & 7 \\ 6 & 9 \end{pmatrix}, \quad \text{where } a \text{ and } b \text{ are constants.}$$

(i) Find $\mathbf{AB} + 2\mathbf{B}$ in terms of a and b.

(ii) If $\mathbf{AB} + 2\mathbf{B} = \mathbf{C}$, find the constants a and b.

(b) The matrices $\mathbf{P}$ and $\mathbf{Q}$ are given by $\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \end{pmatrix}$, and $\mathbf{Q} = \begin{pmatrix} 3 & 4 \\ -1 & 1 \end{pmatrix}$.

(i) Find the matrix $\mathbf{P}^{-1}$.

(ii) If $\mathbf{R}$ is a 2×2 matrix such that $\mathbf{PR} = \mathbf{Q}$, find $\mathbf{R}$ using the inverse matrix obtained in (i).

[3, 2; 3, 3 marks]