MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION EXAMINATION
INTERMEDIATE LEVEL

SEPTEMBER 2015

SUBJECT:	**PURE MATHEMATICS**
DATE:	**4th September 2015**
TIME:	**4.00 p.m. to 7.00 p.m.**

Directions to candidates

Attempt all questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are *not* allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. (a) Find the values of x and y if

$$e^{\ln x + \ln y} = 1,$$
$$xy^2 = 1.$$

(b) Find, correct to three places of decimals, the value of z if

$$2^{5z+1} = 12.$$

(c) Using surds, simplify the expression

$$\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{1-\sqrt{3}}$$

writing your answer in the form $(a + b\sqrt{3})/c$, where a, b, c are integers.

[3; 4; 3 marks]

2. A function $f(x)$ is defined by

$$f(x) = \frac{1}{x^2 + 3x + 2} \equiv \frac{A}{x+1} + \frac{B}{x+2}.$$

   (i) Using partial fractions, obtain a value for the constants A and B.

   (ii) Find $\int_0^2 f(x)dx$.

   (iii) Find $\frac{df}{dx}$ at $x = 0$. **[3, 4, 3 marks]**

3. (a) A quadratic polynomial $p(x)$ is defined by $p(x) = 3x^2 + 6x - 2$. If α and β are the roots of the quadratic equation $p(x) = 0$, find the quadratic equation whose roots are α^2 and β^2.

   (b) The cubic polynomial $q(x)$ is defined by $q(x) = x^3 + ax^2 + bx - 12$, where a and b are constants. This polynomial has factors $x + 3$ and $x - 4$.

   (i) Find the values of a and b.

   (ii) Using division or otherwise find the third factor of $q(x)$.

   (iii) Find the range of $q(x)$ for $x \geq 4$. **[4; 3, 2, 1 marks]**

4. (a) In an arithmetic series, the 8th term is equal to 2.5 times the 3rd term, and the sum of the first ten terms is equal to 87.5. Find the initial term and the nth term.

   (b) In a geometric series, the product of the first three terms is 3375, whilst their sum is 65. Find these three terms.

[5; 5 marks]

5. (a) Using the binomial expansion, find the constants a and b if

$$(1 + ax + bx^2)(1 + 3x)^5 = 1 + 16x + 103x^2 + ...$$

   (b) A small class consists of nine children, 5 boys and 4 girls. A quiz team is to be chosen, consisting of 2 boys and 2 girls.

   (i) In how many different ways can this quiz team be selected from this class?

   (ii) One boy and one girl in this class are twins of each other. How many of the selections in (i) will contain both twins?

   (iii) Find the probability that both twins are selected in the quiz team.

[4; 2, 3, 1 marks]

6. A quadrant of a circle has a radius of 8.5 cm. Find, correct to two places of decimals:

   (i) the area and perimeter of the quadrant;

   (ii) the radius of the semicircle whose area is equal to that of the quadrant;

   (iii) the area of the semicircle whose perimeter is equal to that of the quadrant.

[2, 1; 2, 3 marks]

7. (a) Differentiate the following functions with respect to x:

   (i) $f(x) = e^{2x} \sin 3x$, and

   (ii) $g(x) = x(1 + 3x^2)^4$,
   expressing your answer in the form $p(x)(1+3x^2)^3$, where $p(x)$ is a quadratic polynomial in x.

   (b) A closed rectangular box of dimensions x, $3x$ and y is made of cardboard. Its volume is 900 cm^3.

   (i) Show that its surface area S is given by $S = 6x^2 + \dfrac{2400}{x}$.

   (ii) Find the dimensions of the box which has minimum surface area.

[2, 3; 2, 3 marks]

8. (i) Draw a sketch of the curve $y = \sin x$, for the range $0 \le x \le 2\pi$, indicating on your sketch the coordinates of the turning points, and of the points of intersection of the curve with the x-axis.

   (ii) Find the equation of the line passing through the origin and the point $(\pi/2, 1)$. Draw this line on the sketch in (i).

   (iii) Find by integration the area enclosed by curve in (i) and the straight line in (ii).

[5, 2, 5 marks]

9. (a) Using separation of variables, solve the differential equation

$$\frac{dy}{dx} = \frac{y}{x^2},$$

   given that $y = 1$ when $x = 1$.

   (b) Find the distance from the origin to the tangent at $(1, 1)$ of the function $y = x^2$.

[5, 5 marks]

10. (a) The 2×2 matrix $\mathbf{P}$ represents a reflection in the y-axis, whilst the 2×2 matrix $\mathbf{Q}$ represents a clockwise rotation about the origin through $\pi/2$ radians.

    (i) Write down the matrices $\mathbf{P}$ and $\mathbf{Q}$.

    (ii) Find the matrix product $\mathbf{QP}$, and interpret it geometrically.

    (b) The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{c}$ are given by

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 3 & 2 \\ 2 & -1 & 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 & 1 & 3 \\ -4 & 2 & 1 \\ 7 & -1 & -3 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.$$

    (i) Find the matrix product $\mathbf{AB}$, and hence deduce $\mathbf{A}^{-1}$.

    (ii) Using the method of the matrix inverse, find the vector $\mathbf{x}$ if it satisfies the matrix equation $\mathbf{Ax} = \mathbf{c}$.

[2, 2; 4, 2 marks]