MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD
UNIVERSITY OF MALTA, MSIDA

MATRICULATION CERTIFICATE EXAMINATION
INTERMEDIATE LEVEL
SEPTEMBER 2017

SUBJECT:	PURE MATHEMATICS
DATE:	31st AUGUST 2017
TIME:	16:00 to 19:05

Directions to Candidates

Attempt all questions. There are 10 questions in all and each question carries 10 marks.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

1. (a) (i) Expand $(1+3x)^5$ in ascending powers of x up to and including the term in x^3.
   (ii) Using this expansion, obtain an approximation to 1.01^5 correct to 3 decimal places.
   (iii) Hence, obtain an approximation to 101^5 correct to 4 significant figures.

[1, 2, 1 marks]

(b) The quadratic equation $x^2+5x+13=0$ has roots α and β.
   (i) Show that $\alpha^2+\beta^2=-1$, and hence describe the nature of the roots α and β.
   (ii) Find an equation with integer coefficients whose roots are $\dfrac{\alpha}{\beta}-1$ and $\dfrac{\beta}{\alpha}-1$.

[3, 3 marks]

2. The function $f(x)$ is given by $f(x)=x^3+kx^2+2x+1$.
   (a) Find k, given that $x+1$ is a factor of $f(x)$.

[1 marks]

(b) Factorise $f(x)$ completely.

[3 marks]

(c) The function $g(x)$ is given by

$$g(x)=\frac{1-x}{f(x)}.$$

Express $g(x)$ into partial fractions and find $\int g(x)\,\mathrm{d}x$.

[6 marks]

3. The line ℓ_1 has equation $3y+2x=7$.
   (a) Find the equation of the line ℓ_2 that is parallel to ℓ_1 and passes through the point A with coordinates $(1,3)$.
   [3 marks]
   (b) Find the equation of the line ℓ_3 passing through the points B and C with coordinates $(8,10)$ and $(0,-2)$, respectively. Show that ℓ_3 is perpendicular to ℓ_1.
   [4 marks]
   (c) Find the point of intersection D of the lines ℓ_1 and ℓ_3, and the length AD.
   [3 marks]

4. (a) Use the identity $\cos^2 x+\sin^2 x=1$ to find the value(s) of x between 0^o and 90^o such that $3+3\sin x=4\cos^2 x$.
   [4 marks]
   (b) A function $f(x)$ is defined by $f(x)=x^3-4x$.
      (i) Factorise the expression x^3-4x, and sketch the graph of $f(2x)$. You do **not** need to find the maximum and/or minimum points of this function, but mark clearly the points where the graph intersects the coordinate axes.
      (ii) Find the values of x satisfying the inequality $\dfrac{f(2x)}{8x(x+1)^2}>1$.
      [Hint: First evaluate the expression on the left hand side of the inequality to obtain an expression of the type $\frac{x-a}{x-b}$.]
   [2, 4 marks]

5. (a) The sum to infinity of the geometric progression $a, ar, ar^2, \ldots$ is 3. When the terms of this geometric progression are squared a new geometric progression is obtained whose sum to infinity is $\frac{9}{5}$. Show that $a=3(1-r)$ and hence, find the first term and the common ratio of each geometric progression.
   [5 marks]
   (b) An employee starts work on 1st January 2000 on an annual salary of €40 000. This pay scale will give him an increase of € 700 per annum on the first of January until 1st January 2017 inclusive. He remains on this salary until he retires on 31 December 2040. How much will he earn during his working life?
   [5 marks]

6. (a) Find the coordinates of the stationary point on the curve
$$y=(x^2-4)\sqrt{4x-1}, \quad \text{where } x\geq\frac{1}{4}.$$
   [5 marks]
   (b) Find the equation of the normal to the curve $y=\dfrac{2x-1}{x(x-3)}$ at the point on the curve where $x=2$.
   [5 marks]

7. When a ball is dropped from the roof of a high building, the greatest speed that it can reach is u. One model for its speed v when it has fallen a distance x is given by the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}x} = c\,\frac{u^2 - v^2}{v},$$

where c is a positive constant. Find an expression for v in terms of x given that $v = 0$ when $x = 0$. State what condition we need to put on v with relation to u.

[10 marks]

8. (a) The matrices **B** and **C** are given by

$$\mathbf{B} = \begin{pmatrix} 2 & 4 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{C} = \begin{pmatrix} -4 & 3 \\ -2 & 1 \end{pmatrix}.$$

(i) Find the matrix $\mathbf{B}^{-1}$.

(ii) **Hence**, find the 2×2 matrix **A** such that $\mathbf{AB} = \mathbf{C}$.

[3, 3 marks]

(b) Find the values of the constants x and y in the following matrix equation:

$$\begin{pmatrix} 1 & -1 \\ 0 & 2 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} x \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ y \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}.$$

[4 marks]

9. (a) How many 6–digit even numbers can be formed by using each of the digits $1, 2, 4, 5, 7, 9$ exactly once in each number?

[4 marks]

(b) The students in a class were asked how many pets they have at home. The answers are shown in the table below.

Number of pets	0	1	2	3	4
Number of students	24	15	11	3	2

Find the probability that a student chosen at random from the class has at least two pets.

[2 marks]

(c) All the items produced by two machines during a particular hour were tested. A total of 15% of the items were found to be faulty. It is known that 40% of the total number of items were produced by Machine A. Furthermore, 50% of the total number of items were found to be fully functional (that is, not faulty) **and** produced by Machine B. Find the percentage of the total number of items which were produced by Machine A and found to be faulty.

[4 marks]

10. (a) Find the area of the region bounded by parts of the y-axis, the lines $y = 1$ and $y = 8$ and the curve with equation $y = x\sqrt{x}$.

[4 marks]

(b) Parts of the graphs of $f(x) = 2x^3 + x^2 - 8x$ and $g(x) = 2x^3 - 3x - 4$ enclose a finite region. Find the coordinates of the **two** points of intersection of the two graphs and hence, find the range of values of x for which $g(x) \geq f(x)$. Find the area of this finite region.

[6 marks]