MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL
2018 SECOND SESSION

SUBJECT:	**Pure Mathematics**
DATE:	6^{th} September 2018
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. Three points A, B and C have coordinates $(1,1)$, $(2,5)$ and $(4,3)$.

   (a) Find the equation of the line ℓ_1 passing through the points A and B. (3)

   (b) Find the shortest distance from the point C to the line ℓ_1. (2)

   (c) Find the equation of the line ℓ_2 which passes through C and is perpendicular to the line ℓ_1. (2)

   (d) Using the method of the matrix inverse, find the point of intersection of the lines ℓ_1 and ℓ_2. (5)

[Total: 12 marks]

2. (a) The quadratic equation $x^2+4x-9=0$ has roots α and β. Find $\alpha^2+\beta^2$ and $\alpha\beta$, and hence find the quadratic equation which has roots $\dfrac{\alpha}{\beta}+1$ and $\dfrac{\beta}{\alpha}+1$. (5)

(b) The variables x and y are related by the equation $y = ae^{bx}$, where a and b are constants. The following is a table of values of x and y:

x:	0	2	4	6
y:	2.01	2.45	2.99	3.65

By plotting $\ln y$ against x, find the values of a and b to one significant figure. (5)

[Total: 10 marks]

3. (a) The sum of the first n terms of an arithmetic progression is given by $S_n = \Sigma_{i=1}^{n}(3+10i)$. Find S_n in terms of n, and hence find the least value of n for which $S_n > 1000$. (5)

(b) The number $\lambda = 0.573573573\ldots$ has the digits 573 recurring an infinite number of times. Using geometric progressions, or otherwise, express λ as a fraction in its lowest terms. (3)

[Total: 8 marks]

4. (a) A binomial series is given by $(1-ax)^n = 1-30x+405x^2+\ldots$.

(i) Show that $n=10$ and $a=3$. (3)

(ii) Find the coefficient of x^3 in this series. (1)

(iii) Using a suitable value for x in the above series up to the term in x^3, obtain an approximate value for 0.97^{10}. Find the percentage error in your estimate. (3)

(b) A student has 9 different novels, 3 in Italian, 4 in English and 2 in French on a bookshelf in her study.

(i) In how many ways can the nine novels be arranged on the shelf? (1)

(ii) In how many ways can the nine novels be arranged so that the Italian novels are next to each other? (2)

(iii) What is the probability that in a random arrangement of the nine novels, the Italian novels are next to each other? (1)

(iv) Find the probability that at least one Italian novel is not next to another Italian novel. (1)

[Total: 12 marks]

5. The cubic polynomial $f(x)$ is defined by $f(x) = x^3 + ax^2 + bx + 12$, where a and b are constants. $f(x)$ has $(x+2)$ as one of its factors, and gives a remainder of 12 when divided by $(x-4)$.

   (a) Find the values of a and b. (4)

   (b) Using division, or otherwise, factorise $f(x)$ completely. (2)

   (c) By drawing a rough sketch of $f(x)$, or otherwise, find the values of x for which $f(x) \geq 0$. (2)

[Total: 8 marks]

6. (a) Find the first derivative with respect to x of the functions $f(x)$ and $g(x)$ given by:

$$f(x) = \frac{\ln x}{1 + \sin x}, \quad g(x) = xe^{2x}.$$

   Find the coordinates of the minimum of $g(x)$. (5)

   (b) A rectangular enclosure is bounded on one side by a straight long stream, the other three sides being made up of fencing of lengths x, y and x respectively. A total of 120 metres are available for the fencing of the three sides.

   Find the area of the enclosure in terms of x, and hence find the value of x to maximise the area of the enclosure. Calculate the maximum area. (5)

[Total: 10 marks]

7. (a) A hot teapot cools by convection to the outside air. Its temperature is given by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -2(y - 20),$$

   where $y(t)$ is the temperature in degrees Centigrade at time t hours after $t = 0$. Initially, at time $t = 0$, $y = 100$.

   (i) By separating the variables and integrating, find y in terms of t. (6)

   (ii) Find t when $y = 40$. (1)

   (b) Using surds, find integers a, b and c if $\dfrac{2\sqrt{3}-1}{\sqrt{3}+2} \; \dfrac{a\sqrt{3}+b}{c} = \dfrac{6\sqrt{3}+2}{2\sqrt{3}+3}$. (3)

[Total: 10 marks]

8. (a) Using the identity $\sin^2 x + \cos^2 x = 1$, or otherwise, find the three values of x in the range $0 \le x \le \pi$, which satisfy the equation $1 - \sin x = \cos^2 x$. (3)

(b) A function is defined by $h(x) = \sin\frac{x}{2} - 1$.

(i) Draw a sketch of $y = h(x)$ for the range $0 \le x \le 2\pi$. (3)

(ii) Find the integral $\int_0^{2\pi} h(x)\mathrm{d}x$. (2)

[Total: 8 marks]

9. (a) The matrix **A** is defined by $\mathbf{A} = \begin{pmatrix} -1 & 3 \\ -2 & 7 \end{pmatrix}$.

(i) Find the matrix **X** such that $\mathbf{AX} = \mathbf{I}$. What is **X** called? (2)

(ii) Show that $\mathbf{X}^2 + 6\mathbf{X} - \mathbf{I} = \mathbf{0}$. (3)

(b) The matrix **B** represents a clockwise rotation of $\pi/2$.

(i) Write down the matrix **B**. (1)

(ii) Find $\mathbf{B}^2$ and $\mathbf{B}^4$. (2)

(iii) Interpret the matrices in part (ii) geometrically. (2)

[Total: 10 marks]

10. (a) A circle has a radius of 1 metre. An equilateral triangle is drawn with its vertices lying on the circumference of the circle.

Find the area of any one of the segments bounded by one side of the triangle and the shorter part of the circumference of the circle. (4)

(b) A function f(x) is defined by $f(x) = \dfrac{4}{4 - x^2}$.

(i) Express $f(x)$ in terms of partial fractions. (3)

(ii) Find the equation of the tangent to the graph of $y = f(x)$ at $x = 1$. (5)

[Total: 12 marks]