MATRICULATION AND SECONDARY EDUCATION CERTIFICATE
EXAMINATIONS BOARD

L-Università ta' Malta

INTERMEDIATE MATRICULATION LEVEL
2019 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	4th May 2019
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

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1. (a) The line ℓ_1 passes through the points $(1,0)$ and $(0,1)$.
   (i) Find the equation of the line ℓ_1. (2)
   (ii) Find the perpendicular distance of this line from the origin. (2)
   (iii) Show that the line ℓ_2 defined by $y = x$ is perpendicular to ℓ_1. (1)
   (iv) Find the point of intersection of ℓ_1 and ℓ_2. (1)

   (b) The function $f(x)$ is defined by

$$f(x) = \frac{2x+2}{x(x+2)} \equiv \frac{A}{x} + \frac{B}{x+2}.$$

   (i) Find the values of A and B in the partial fraction representation of $f(x)$. (2)
   (ii) Find $\int_1^2 f(x)dx$. (2)

[Total: 10 marks]

2. The quadratic $p(x)$ is defined by $p(x) = x^2 + 2x + 3$.

   (a) By completing the square, or otherwise, find the coordinates of the minimum of the graph of $y = p(x)$. Find the range of $p(x)$ (3)

   (b) Find the equation of the tangent of the graph of $y = p(x)$ at $x = 2$. (3)

   (c) The roots of the quadratic equation $p(x) = 0$ are denoted by α and β.

      (i) Find $\alpha^2 + \beta^2$, and deduce the nature of the roots. (2)

      (ii) Find the quadratic equation whose roots are $5\alpha + \beta$ and $\alpha + 5\beta$. (3)

[Total: 11 marks]

3. (a) Using arithmetic progressions, find the sum of the integers between 1 and 200, both inclusive, but excluding all multiples of 3. (4)

   (b) Using geometric progressions, find the initial term, the common ratio and the sum to infinity, S, of the series defined by

$$S = \sum_{r=0}^{\infty} e^{1+xr},$$

   assuming that the sum exists. If $S = 5$, find the value of x to four places of decimals.

   (5)

[Total: 9 marks]

4. (a) (i) Using Pascal's triangle or otherwise, express $(x+7)^3 - (x-7)^3$ as a quadratic in x. (3)

      (ii) Hence find the exact value of $(\sqrt{50}+7)^3 - (\sqrt{50}-7)^3$. (1)

      (iii) Using the fact that $0 < \sqrt{50} - 7 < 1/10$, show that $(\sqrt{50}+7)^3$ differs from an integer by less than $1/1000$. (1)

   (b) Five-digit numbers are to be formed from the five digits 1, 2, 4, 7, 8, with no digit being repeated within each number.

      (i) How many different numbers can be formed? (1)

      (ii) How many odd numbers can be formed? (2)

      (iii) If the digit selection is random, what is the probability that an odd number is formed? (1)

      (iv) What is the probability that an even number is formed? (1)

[Total: 10 marks]

5. (a) Using the identity $\sin^2 x + \cos^2 x = 1$, or otherwise, find two values of x in the range $0 \leq x \leq 2\pi$, which satisfy the equation $6\sin^2 x - 7\cos x - 1 = 0$. (3)

   (b) A circle has a radius of 1 cm. A regular pentagon is inscribed in the circle. Find to four places of decimals, the area and perimeter of a minor segment formed by the circle and one side of the pentagon. (7)

[Total: 10 marks]

6. (a) Find the first derivative with respect to x of the functions:

   (i) $f(x) = \dfrac{1+x}{1+x^2}$, (2)

   (ii) $g(x) = x(1+2x)^5$, giving your answer in the form $(ax+b)(1+2x)^4$, where a and b are integers. (2)

   (b) A closed circular cylinder is made of thin cardboard, and has radius r, height h and volume 100π cm^3. Find h in terms of r, and express the surface area S of the cylinder in terms of r only. By differentiating with respect to r, find to the nearest cm^2 the minimum surface area of the cylinder. (6)

[Total: 10 marks]

7. (a) A differential equation is given by $\dfrac{dy}{dx} = \cos\left(\dfrac{x}{2}\right)$, given that $y = 0$ when $x = 0$.

   Solve this system by separating the variables. Draw, on the same graph, a sketch of y and $\dfrac{dy}{dx}$ in the range $0 \leq x \leq 4\pi$. (5)

   (b) Find the coordinates of the points where the curve $y = x^2 - x - 6$ intersects the x-axis. Hence, find by integration, the area enclosed by this curve and the x-axis.

   (5)

[Total: 10 marks]

8. The cubic polynomial $q(x)$ is defined by $q(x) = x^3 + ax^2 + bx + 3$, where a and b are constants. The polynomial $q(x)$ has $x+3$ as factor, and leaves a remainder of 16 when it is divided by $x-1$.

   (a) Find a and b. (4)

   (b) Factorise $q(x)$. (3)

   (c) Draw a sketch of the graph of $y = q(x)$. (2)

   (d) For which values of x is $q(x) \geq 0$? (1)

[Total: 10 marks]

9. (a) Find the values of x and y if

$$e^{\ln x+\ln y}=2,$$
$$x^2y=1.$$ (3)

(b) Find the value of x if $3^{2x+1}4^{1-x}=\dfrac{16}{3}$. (3)

(c) Using surds, simplify the expression $\dfrac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}$. Express it in the form $a-b\sqrt{6}$, where a and b are integers to be determined. (3)

[Total: 9 marks]

10. (a) Find the matrix **A** which represents an anti-clockwise rotation through $\pi/2$. (2)

(b) The matrix **B** is given by $\mathbf{B}=\begin{pmatrix}1 & 2\\ -3 & 4\end{pmatrix}$.

(i) Find the determinant and inverse of **B**. (4)

(ii) Calculate $\mathbf{B}^2-6\mathbf{B}+2\mathbf{I}$, where **I** is the 2 x 2 identity matrix. (3)

(iii) Using the method of the matrix inverse, solve the system of equations

$$\begin{pmatrix}1 & 2\\ -3 & 4\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}20\\ 30\end{pmatrix}.$$ (2)

[Total: 11 marks]