L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL

2019 SECOND SESSION

SUBJECT:	**Pure Mathematics**
DATE:	5th September 2019
TIME:	9:00 a.m. to 12:05 p.m.

Directions to candidates

Attempt **ALL** questions. There are 10 questions in all.

The marks carried by each question are shown at the end of the question.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

---

1. (a) Find the values of x and y if

$$3\log_2 x = y,$$

$$\log_2(4x) = y - 3.$$

(4)

(b) Two variables x and y are related by a law of the type $y = ab^x$, where a and b are constants. The following values of x and y were recorded in a physical experiment:

x:	1	2	3	4	5
y:	9.0	13.5	20.4	30.0	45.9

By plotting $\ln y$ against x, find the values of a and b to one decimal point. (6)

[Total: 10 marks]

2. (a) Using surds, find the integers a and b if $\dfrac{\sqrt{3}+1}{2\sqrt{3}+3}(a\sqrt{3}+b)=\dfrac{\sqrt{3}-1}{2-\sqrt{3}}$. (3)

(b) The function $f(x)$ is defined by

$$f(x)=\frac{1}{(x+1)(2x+1)}\equiv\frac{A}{x+1}+\frac{B}{2x+1}.$$

(i) Find the values of A and B in the partial fraction representation of $f(x)$. (3)

(ii) Find $\int_0^1 f(x)dx$. (4)

[Total: 10 marks]

3. A and B are the points (0,3) and (4,0) respectively.

(a) Find the equation of the line AB. (3)

(b) Find the equation of the line passing through (9,12) and perpendicular to AB. (3)

(c) Find the coordinates of the point of intersection of the lines in (a) and (b). (2)

(d) Find the distance of the line AB from the point (1,2). (2)

[Total: 10 marks]

4. (a) The quadratic $p(x)$ is defined by $p(x)=x^2+ax+b$, where a and b are constants. $p(x)$ gives a remainder of 2 when divided by $x+1$, and a remainder of 27 when divided by $x-4$. Find a and b. (3)

(b) If the roots of the quadratic equation $x^2+2x+3=0$ are denoted by α and β, find the quadratic equation whose roots are $2\alpha+2\beta$ and $3\alpha\beta$. (3)

(c) Using integration, find the area enclosed by the curve $y=5+\sin 2x$, the x-axis, and the lines $x=0$ and $x=\pi$. (4)

[Total: 10 marks]

5. (a) Find the least number of terms of the arithmetic series $5+12+19+\cdots$ needed to make a sum exceeding 2000. (5)

(b) The sum to infinity of a convergent geometric series is equal to the sum to infinity of the squares of its terms. If the first term of the geometric series is $\frac{3}{2}$, find its common ratio. (5)

[Total: 10 marks]

6. (a) A binomial expansion is given by $(1+ax)^n = 1+12x+54x^2+\cdots$, where a and n are integers.

   (i) Find the values of the integers a and n. (5)

   (ii) Find the coefficients of the terms in x^3 and x^4 in the above expansion. (2)

   (b) The transformation $\mathbf{R}$ is defined as a reflection in the x-axis.

   (i) Find $\mathbf{R}$. (1)

   (ii) Find $\mathbf{R}^2$. What is the geometric significance of this result? (2)

[Total: 10 marks]

7. (a) The matrix $\mathbf{B}$ is given by $\mathbf{B} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$, and $\mathbf{I}$ is the identity 2×2 matrix.

   Find the values of a and b if $\mathbf{B}^2 = a\mathbf{B} + b\mathbf{I}$. (3)

   (b) Solve for x if $\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} x & 1 \\ 1 & x \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 14$. (2)

   (c) Using the method of the matrix inverse, solve the system of equations

$$\begin{pmatrix} 1 & 3 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 11 \\ 16 \end{pmatrix}.$$ (5)

[Total: 10 marks]

8. (a) A circle of radius of 1 cm has centre O, with P being a point on its circumference. Another circle of radius 1 cm is drawn, with centre at P. Find the area which is common to both circles. (6)

   (b) An equation is given by $\sin x = \frac{1}{2}$, with x being restricted to the range $0 \le x \le 2\pi$.

   (i) Find the values of x which satisfy this equation. (2)

   (ii) Draw a sketch of the curve $y = \sin x$ in the range $0 \le x \le 2\pi$, and mark the solutions obtained in part (b) (i) on this sketch. (1)

   (iii) Find the range of values of x for which $2\sin x - 1 \ge 0$. (1)

[Total: 10 marks]

9. (a) A bag contains 4 white and 7 black balls. Apart from the colour, the balls are identical. A ball is chosen at random from the bag and its colour is recorded. This ball is **not** placed back in the bag. A second ball is then chosen at random from the bag, and its colour is recorded.

   (i) Draw a tree diagram to describe the situation described above. (3)

   (ii) Find the probability that both balls are of the same colour. (2)

   (iii) Find the probability that the balls extracted have different colours. (1)

   (b) A union official wants to elect a committee of 4 members. In all, there are 11 candidates, of which 6 are academics, and 5 are non-academics. Find:

   (i) the number of ways in which this committee is selected from all the candidates; (1)

   (ii) the number of ways in which only academics are selected; (1)

   (iii) the probability that the committee consists only of academics; (1)

   (iv) the probability that the committee contains at least one non-academic. (1)

[Total: 10 marks]

10. (a) Find the first derivative with respect to x of the functions:

   (i) $f(x) = (1+x)\sin x$; (2)

   (ii) $g(x) = \dfrac{e^{2x}}{1+e^{2x}}$. (2)

   (b) A thin wire of length 4 cm is bent to form an isosceles triangle. Taking the base of the triangle to be of length $2x$, show that the area of the triangle is

$$A(x) = 2x(1-x)^{1/2}.$$

   Hence find by differentiation, the length of the sides of the triangle when the area is maximum. (6)

[Total: 10 marks]