



SUBJECT:	Pure Mathematics
DATE:	7 th May 2022
TIME:	9:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.

The total number of marks for all the questions in the paper is 100.

Graphical calculators are **not** allowed.

Scientific calculators can be used, but all necessary working must be shown.

A booklet with mathematical formulae is provided.

1. Express

$$f(x) = \frac{3x^2 - 14x + 22}{(x-5)(x-2)^2}$$

into partial fractions and show that:

$$\int_{10}^{15} f(x) dx = \ln 8 - \frac{5}{52}.$$

[Total: 8 marks]

2. (a) Solve the equation $25^x - 9 \times 5^x + 20 = 0$.

[3 marks]

(b) Find the exact solution of the equation $2^{3x-4} \times 5^{2x-5} = 100^{x-2}$, giving your answer in the form $\frac{\ln p}{\ln q}$ where p and q are integers.

[4 marks]

(c) In a physics experiment, Nick measured how the force, F , exerted by a spring depends on its extension, x . He then plotted the values $A = \ln F$ and $B = \ln x$ on a graph, with B on the horizontal axis and A on the vertical axis. The graph he obtained was a straight line, passing through the points $(2, 4.5)$ and $(4, 7.2)$. Find F when $x = 2.5$.

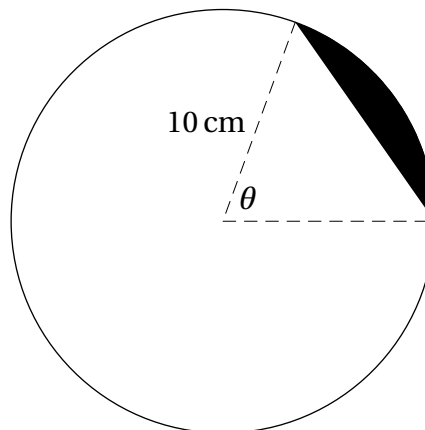
[5 marks]

[Total: 12 marks]

3. (a) Write the expression $4x^2 - 10x - 9$ in the form $a(x+h)^2 + k$ and hence find the exact solution of the equation $4x^2 - 10x - 9 = 0$, giving your answer in surd form. [4 marks]
- (b) The coefficient of x^2 in the expansion of $(1 + ax)^n$ is 54 and the coefficient of x is 12. Find the values of a and n . [4 marks]
- (c) An arithmetic progression consists of 100 terms. The third term is -26 and the tenth term is 58. Find the first term, the common difference and the sum of the terms of the progression. [4 marks]

[Total: 12 marks]

4. (a) Given that the area of the shaded region in the following diagram is 20.55 cm^2 , show that $\theta - \sin \theta = 0.411$.



[3 marks]

- (b) Draw the graphs $y = \cos x$ and $y = \sin x$ for $0 \leq x \leq 2\pi$. On the graphs indicate clearly the parts where x is obtuse. [3 marks]
- (c) Let x be an obtuse angle and let $k = \sin x$. Find $\cos x$ in terms of k . Hence, or otherwise, solve the following equation.

$$\sin x - \sqrt{3} \cos x = \sqrt{3}$$

[4 marks]

[Total: 10 marks]

5. Show that:

$$\frac{4x^2 + 2x - 9}{2x + 3} = 2x - 2 - \frac{3}{2x + 3},$$

and solve the differential equation

$$(2x + 3)\frac{dy}{dx} = y(4x^2 + 2x - 9),$$

given that $y = 1$ when $x = -1$. In your answer express y as a function of x .

[Total: 6 marks]

6. Differentiate the following functions with respect to x .

(a) $f(x) = 3 \cos^3 x + 2x$

(b) $g(x) = x^4 e^{3x}$

(c) $h(x) = \frac{x^2 + 5x}{\ln x}$

[2, 3, 3 marks]

[Total: 8 marks]

7. The line ℓ_1 has equation $y = -2x + 6$. The line ℓ_2 passes through $A(5, 6)$ and is perpendicular to ℓ_1 .

(a) Find the equation of ℓ_2 .

[2 marks]

(b) Find the coordinates of the point B where ℓ_1 and ℓ_2 intersect.

[2 marks]

(c) Find the distance AB , giving your answer in the simplest surd form.

[2 marks]

(d) Let R denote the x -intercept of ℓ_1 . Show that triangle ABR is isosceles and find its area.

[4 marks]

[Total: 10 marks]

8. (a) For the curve $y = x^3 + 6x^2 + 9x$ find the coordinates of the minimum and maximum turning points and distinguish between them.

[6 marks]

(b) On the same set of axes, sketch the graphs of the curve $y = x^3 + 6x^2 + 9x$ and the line $y = x$, showing clearly the coordinates of the points where the two graphs intersect.

[4 marks]

(c) Hence, find the area enclosed between the curve and the line.

[4 marks]

[Total: 14 marks]

9. A group of 7 athletes consists of 3 girls: Amy, Beth and Caroline, and 4 boys: Daniel, Emanuel, Franco and George.

(a) The 7 athletes are to be seated next to each other on a bench.

(i) In how many ways can the athletes be seated?

(ii) In how many ways can the athletes be seated if Franco must sit in the middle?

(iii) In how many ways can the athletes be seated if the girls must sit next to each other?

[1, 2, 2 marks]

(b) 4 of the 7 athletes are to be chosen to participate in a competition.

(i) In how many ways can this be done?

If the athletes are chosen at random, what is the probability that:

(ii) they are **all** boys?

(iii) there are more boys than girls?

[1, 2, 2 marks]

[Total: 10 marks]

10. (a) Let $\mathbf{P} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 4 \\ 5 & 6 & 6 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -18 & 6 & 5 \\ 26 & -9 & -7 \\ -11 & 4 & 3 \end{pmatrix}$.

(i) Find \mathbf{PQ} .

(ii) What do you deduce about the matrices \mathbf{P} and \mathbf{Q} ?

(iii) Use your result to solve the following equations.

$$\begin{aligned} x + 2y + 3z &= -1 \\ -x + y + 4z &= 1 \\ 5x + 6y + 6z &= -1 \end{aligned}$$

[1, 1, 3 marks]

(b) The 2×2 matrix \mathbf{A} represents a reflection in the line $y = x$. The 2×2 matrix \mathbf{B} represents an anticlockwise rotation by 90° about the origin.

(i) Find the transformation matrices \mathbf{A} and \mathbf{B} .

(ii) Give a geometric description of the transformation represented by the matrices \mathbf{A}^2 , \mathbf{B}^3 and \mathbf{AB} .

[2, 3 marks]

[Total: 10 marks]