L-Università ta' Malta

MATRICULATION AND SECONDARY EDUCATION CERTIFICATE EXAMINATIONS BOARD

INTERMEDIATE MATRICULATION LEVEL
2023 FIRST SESSION

SUBJECT:	**Pure Mathematics**
DATE:	6th May 2023
TIME:	9:00 a.m. to 12:05 p.m.

Directions to Candidates

Answer **ALL** questions. There are **10** questions in all.
The total number of marks for all the questions in the paper is 100.
Graphical calculators are **not** allowed.
Scientific calculators can be used, but all necessary working must be shown.
A booklet with mathematical formulae is provided.

1. (a) Solve $x(3x-2)-3x(2-x)+2(x+4)=x+6$.

[3 marks]

(b) Find the range of values of k for which $k^2-4k>0$.

[3 marks]

(c) Find the set of values of k for which the line $y=kx-k$ intersects the curve $y=x^2$ at two distinct points.

[2 marks]

[**Total: 8 marks**]

2. Let $f(x)=2x^3-x^2-8x+4$.

(a) Show that $x-2$ and $x+2$ are factors of $f(x)$. Find the other factor of $f(x)$.

[3 marks]

(b) Express into partial fractions

$$\frac{x^2-30x-4}{2x^3-x^2-8x+4}.$$

[4 marks]

(c) Hence, show that

$$\int_3^8 \frac{x^2-30x-4}{2x^3-x^2-8x+4}\,\mathrm{d}x = -2\ln 2-\frac{5}{2}\ln 3.$$

[5 marks]

[**Total: 12 marks**]

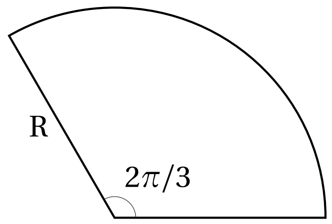
3. (a) Rationalise the denominator of $\dfrac{2\sqrt{2}-\sqrt{3}+\sqrt{6}-2}{\sqrt{2}-1}$.

[3 marks]

(b) Solve the equation $4^x\,5^{2x} = 9$ giving your answer in the form $x = \log_{10} a$ for an integer a.

[3 marks]

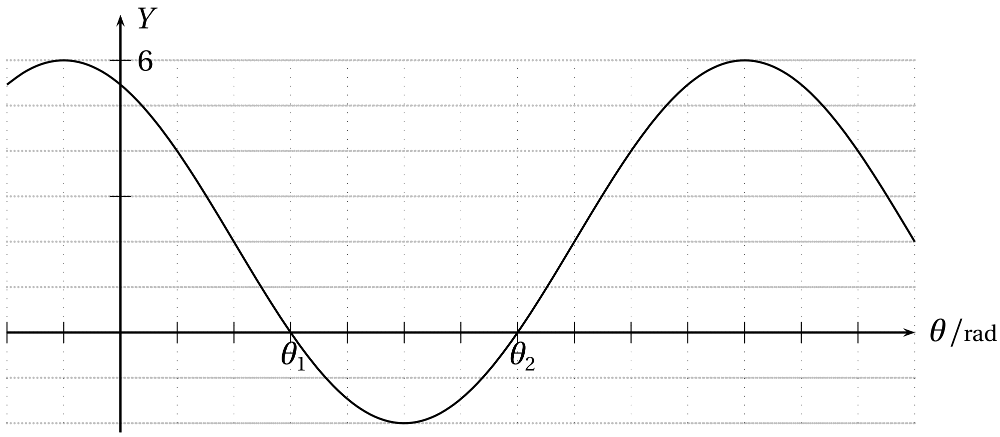
(c) A carpenter makes a wooden top for a table in the shape of a sector of a circle, as shown in the figure below. If the angle at the centre is $\frac{2\pi}{3}$ rad and the area of the wooden top is 3π m^2, find the radius R, and the perimeter of the wooden top.



[2, 2 marks]

[**Total: 10 marks**]

4. The diagram shows the graph of $Y = Y_0 + 4\cos\left(\theta + \frac{\pi}{6}\right)$, where θ is in radians and Y_0 is a constant.



(a) In terms of Y_0, θ_1 and θ_2, find the area that is enclosed under the horizontal axis by the graph of $Y = Y_0 + 4\cos\left(\theta + \frac{\pi}{6}\right)$.

[4 marks]

(b) Use the diagram and the characteristics of the cosine function to find the values of Y_0, θ_1 and θ_2.

[3 marks]

(c) Show that the value of the area found in part (a) is equal to $4\left(\sqrt{3}-\frac{\pi}{3}\right)$.

[3 marks]

[**Total: 10 marks**]

5. (a) The fifth term of an arithmetic progression is 32. The tenth term is 67.
   (i) Find the first term and the common difference.
   (ii) Which term first exceeds 5000?

[3, 2 marks]

(b) The first term of a geometric progression is 50. The sum of the second and third terms is 12.
   (i) Find the possible values of the common ratio.
   (ii) Find the sum to infinity in the case where it exists.

[3, 2 marks]

[**Total: 10 marks**]

6. Two variables x and y are related to one another by the equation $y = a\,b^x$, where a and b are positive constants. The table below gives a few corresponding values of x and y.

x-value	0.5	1	1.5	2	2.5	3
y-value	4	7	14	27	50	95

(a) By taking logarithms (to base 10), reduce the given equation into linear form.

[1 mark]

(b) On the graph paper provided, plot a graph of $\log_{10} y$ against x.

[5 marks]

(c) Find the gradient of the resulting line and write down the value of the vertical-intercept. Use these values and the equation deduced in part (a) to find the values of a and b. Round the value of a to the nearest whole number and the value of b to one decimal place.

[4 marks]

[**Total: 10 marks**]

7. The following three matrices, representing linear-transformations of the xy-plane, satisfy the equation $AB = C$.

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(a) Give a geometric description of the transformations represented by A and C.

[2 marks]

(b) Find the values of a, b, c and d and describe the transformation B geometrically.

[3 marks]

(c) Show that $AB + BA = 0$.

[2 marks]

(d) Find $(BA)^{-1}$.

[1 mark]

[**Total: 8 marks**]

8. (a) Find the equation of the normal to the curve $y = \dfrac{3}{\sqrt{2x+1}}$ at the point where $x = 4$.

[4 marks]

(b) Oil is poured onto a table, forming a circle whose area increases at a rate of 2.5 $\text{cm}^2\ \text{s}^{-1}$. Find the rate the radius is increasing when the area of the circle is 20π cm^2.

[4 marks]

(c) Find the stationary points on the curve given by equation $y = \dfrac{x^2 - x + 1}{1 - x}$, and determine their nature.

[6 marks]

[**Total: 14 marks**]

9. (a) Work out the following integral.

$$\int \frac{x}{x^2 - 3}\,\mathrm{d}x$$

[3 marks]

(b) Hence, by separating the variables, find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xy^2}{x^2 - 3},$$

given that $y = -\dfrac{1}{2}$ when $x = 2$.

[5 marks]

[**Total: 8 marks**]

10. Amie has a collection of twelve **different** soft-toys: six dogs, two bears and four elephants.

(a) Amie decides to put all her twelve soft-toys next to each other on a shelf. In how many ways can she do this if:

(i) there is no restriction on the way they are placed?
(ii) the elephants must be placed next to each other?
(iii) there must be one bear at each end of the shelf?

[1, 2, 2 marks]

(b) Amie decides to let her sister Linda have three of her soft-toys. In how may ways can Linda choose these three soft-toys if:

(i) she can choose any three?
(ii) she is allowed to take one dog, one bear and one elephant?
(iii) she is not allowed to take any of the bears?

[1, 2, 2 marks]

[**Total: 10 marks**]