AM SYLLABUS (2012)

PURE MATHEMATICS AM 27

SYLLABUS

Pure Mathematics AM 27	(Available in September)
Syllabus	Paper I(3hrs)+Paper II(3hrs)

1. AIMS

- To prepare students for further studies in Mathematics and related subjects.
- To extend the students' range of mathematical techniques so as to apply them in more difficult and unstructured problems.
- To develop in students the ability to read and understand a wider range of mathematical articles and arguments.
- To enable students to formulate a mathematical representation of a real life situation.
- To use appropriate technology such as computers and calculators as a mathematical tool.
- To encourage confidence, enjoyment and satisfaction through the development and use of Mathematics.

The syllabus assumes a good knowledge of the subject at SEC level and coverage of the extension topics in Paper 2A. It aims at consolidating this knowledge and to extend it to include more advanced concepts.

2. ASSESSMENT OBJECTIVES

Candidates are required to:

- demonstrate their knowledge of mathematical facts, concepts, theories and techniques in different contexts.
- construct mathematical arguments and proofs by means of precise statements, logical deduction and inference.
- recognise standard models and be able to apply them.

3. SCHEME OF ASSESSMENT

The examination will consist of 2 papers of 3 hours each. Any examination question can test material from more than one topic. Questions may be set on topics which are not explicitly mentioned in the syllabus but such questions will contain suitable guidance so that candidates will be able to tackle them with the mathematical knowledge they would have acquired during their studies of the material in the syllabus. Knowledge of topics in Paper 1 is assumed and may be tested in Paper 2.

Scientific calculators may be used but all necessary working must be shown. The use of graphical calculators is however prohibited. Disciplinary action will be taken against students making use of such calculators. A booklet with mathematical formulae will be provided.

Paper 1 will contain 10 questions, possibly of varying difficulty. Marks allotted to each question will be shown. The total number of marks available in the paper is 100 and candidates will have to answer all the questions.
Paper 2 will contain 10 questions and candidates will be asked to choose 7 questions. Each question will carry 15 marks.

4. GRADE DESCRIPTION

Grade A:

- Candidates who are able to recall and select almost all concepts, techniques and theories required in different contexts.
- Candidates who use diagrams and sketches with a high level of accuracy and who are able to proceed logically in their proofs.
- Candidates who derive results to a high degree of accuracy.

Grade C:

- Candidates who are able to recall and select most concepts, techniques and theories required in different contexts.
- Candidates who use diagrams and sketches with a reasonable level of accuracy and who are able to proceed logically in their proofs.
- Candidates who derive results to an appropriate degree of accuracy.

Grade E:

- Candidates who are able to recall and select some concepts, techniques and theories required in different contexts.
- Candidates who use diagrams and sketches with some accuracy and who are able to proceed logically in their proofs.
- Candidates who derive results to a fair degree of accuracy.

5. SUBJECT CONTENT

The topics are not arranged in teaching order. The syllabus is not meant as a teaching scheme and teachers are free to adopt any teaching sequence that they deem to be suitable for their students.

Paper 1

	Topics	Notes
1.	Polynomials, rational functions, the factor and remainder theorems.	Simplification of rational expressions including factorising and cancelling, and algebraic division.
	Positive and negative rational indices. Simple partial fractions.	Problems on partial fractions could include denominators such as: $(ax+b)(cx+d)(ex+f)$, $(ax+b)(cx+d)^2$ and $(ax+b)(cx^2+dx+e)$. The degree of the denominator must not be greater than *three.*
	Laws of indices. Laws of logarithms. Use and manipulation of surds.	
	The quadratic equation in one variable.	Knowledge of the relation between the roots α, β and the coefficients of a quadratic equation. Forming new equations with roots related to the original. Calculation of expressions such as $\alpha^3+\beta^3$.
	The quadratic function.	Method of completing the square.

	Topics	Notes
	Simple inequalities in one variable	Graphical or algebraic solution of inequalities such as the following: (i) $(x-a)(x-b)(x-c)>0$, (ii) $(x-a)/(x-b)<c$, (iii) $\lvert\cos x\rvert \le 0.4, \quad 0 \le x \le \pi$.
2.	Arithmetic series, finite and infinite geometric series	The general term and the summation of the geometric and arithmetic series. Arithmetic and geometric mean. Knowledge of the notations $\sum$ and $\binom{n}{r}$.
	The binomial expansion for rational indices including the range of convergence.	
3	Simple counting problems involving permutations and combinations. Applications to simple problems in probability.	The knowledge of probability expected will be limited to the calculation of probabilities arising from simple problems of enumeration of equally likely possibilities, including simple problems involving the probability of the complement of an event and of the union and intersection of two events.
4.	Plane Cartesian coordinates. Simple curve sketching.	Curve sketching will be limited to polynomials up to three stationary points. Knowledge of the effect of simple transformations on the graph of $y = f(x)$ as represented by $y = f(x+a)$, $y = f(x)+a$, $y = f(ax)$, $y = af(x)$, and combinations of these transformations. The relation of the equation of a graph to its symmetries.
	The coordinates of a point on a curve in terms of a parameter. Distances, angles, elementary treatment of lines, circles and curves given by simple Cartesian or parametric equations.	Knowledge of specific properties of curves other than the straight line and the circle will not be required.
5.	Functions. One – one and onto functions. Inverses for one – one functions and the graphical illustration of the relationship between a function and its inverse. Composition of functions. Modulus of a function	Students should be able to demonstrate a clear understanding of what is meant by the domain, codomain and the range of a function. Inverses of composite functions will not be required.

	Topics	Notes
	The exponential and logarithmic functions and their graphs.	The exponential and logarithmic functions as inverse functions of each other.
6.	The trigonometric functions and their inverses for any angle and their graphs. Trigonometric identities, including compound angle, double angle and half angle identities. Use of factor formulae.	Manipulative skills are expected but questions requiring lengthy manipulations will not be set. Questions on how to prove the addition theorems will not be set.
	Solutions of simple trigonometric equations.	Knowledge of the general solutions.
	Radian measure. Use of the formulae: $s = r\theta$, $A = \frac{1}{2}r^2\theta$. The approximations $\sin x \approx x \approx \tan x$, $\cos x \approx 1 - \frac{x^2}{2}$. Transformation of the expression $a\cos\theta + b\sin\theta$ into the forms such as $R\cos(\theta - \alpha)$.	Knowledge of the values of cosine, sine and tangent of $\frac{\pi}{k}$, where $k = 1, 2, 3, 4, 6,$ in surd or rational form.
7.	Complex numbers. The forms $a + ib$ and $r(\cos\theta + i\sin\theta)$. Modulus, argument, conjugates. The Argand diagram. Sums, products and quotients. Equating real and imaginary parts. Simple examples of conjugate roots of polynomials with real coefficients.	The argument satisfies $-\pi < \theta \le \pi$.
8.	The derivative as a limit. Differentiation of sums, products, quotients and composition of functions.	A rigorous treatment of limits is not expected.
	The differentiation of algebraic, trigonometric, exponential and logarithmic functions. Differentiation of simple functions defined implicitly or parametrically.	Inverse trigonometric functions are excluded.
	Applications of differentiation to gradients, tangents and normals. Simple problems on maxima and minima, and curve sketching.	

	Topics	Notes
9.	Integration as the limit of a sum and as the inverse of differentiation.	A rigorous treatment is not expected.
	The evaluation of integrals by means of standard forms, by substitution, by partial fractions and by parts.	The ability to use trigonometric identities to integrate functions.
	Applications of integration to the calculation of areas and mean values of functions.	
10.	First order differential equations with separable variables.	
11.	Vectors in two and three dimensions. Use of the unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$.	Vectors represented by $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ may be represented by $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$.
	Addition, subtraction and multiplication by a scalar, and their geometric interpretation.	
	The equation of a line in the form $\mathbf{r}=\mathbf{a}+t\mathbf{b}$.	
	The scalar product of two vectors.	Intuitive understanding of skew lines in three dimensions.
	Intersecting lines in three dimensions.	
12.	The algebra of matrices. Addition and multiplication. Distributivity of multiplication over addition. Associativity. The zero matrix and the identity matrix. Non commutativity of multiplication. The inverse of a matrix.	Students will only be expected to be able to find inverses of 2×2 matrices; but they should be able to verify that, say two given 3×3 matrices are inverses of each other.
	Linear transformations in the plane.	The ability to find the matrix associated with a linear transformation and vice-versa, for example, rotation through angle θ about the origin and reflection in the line $y=x\tan\theta$. Understand composition of transformations and the connection with matrix multiplication.

Paper 2

	Topics	Notes
1.	Summation of simple finite series.	Simple applications, for example, the method of differences and the sum of the squares and cubes of the first n natural numbers.
	Summation of simple infinite series.	Summation of infinite series by finding the limit of the sum to n terms and also by the use of binomial, logarithmic, exponential, trigonometric and hyperbolic series. Knowledge of the region of convergence.
	Maclaurin series.	The ability to find Maclaurin series of a function, including the general term in simple cases.
2.	Method of induction.	Proof of De Moivre's theorem (for positive integer index only). Summations of series, finding general forms in a sequence and divisibility. Use of induction in inequalities and equations involving matrices.
3.	The exponential form of a complex number. The n n^{th} roots of a complex number.	Know that in the Argand diagram the n n^{th} roots of a non-zero complex number are the vertices of a regular n sided polygon.
	De Moivre's Theorem for any rational index, and simple applications.	Applications of De Moivre's Theorem to the derivation of trigonometric identities.
	The exponential form for $\sin\theta$ and $\cos\theta$.	
	Equations and inequalities involving the modulus sign.	$\lvert z-z_1\rvert \le a$, $\lvert z-z_1\rvert \le k\lvert z-z_2\rvert$ and their corresponding loci and region in the Argand diagram.
4.	Definition and properties of the vector product. Triple scalar product. The equation of a plane in the form $\mathbf{r}\cdot\mathbf{n}=a$.	Knowledge of the triple vector product is not required.
	Applications of vectors to two and three dimensional geometry, involving points, lines and planes.	Area of a parallelogram and triangle, volume of parallelepiped and tetrahedron.
	Cartesian coordinate geometry of lines and planes. Direction ratios.	

	Topics	Notes
5.	Odd, even and periodic functions. Further sketching of graphs including those of rational functions such as $\dfrac{ax^2+bx+c}{px^2+qx+r}$.	Treatment and sketching of graphs of the form $y = f(x)$, $y^2 = f(x)$. The use of a graph to determine the range of a function. The ability to obtain the graph of the reciprocal of a function from a graph of the function.
	Asymptotes.	Symmetry asymptotes, including oblique asymptotes.
6.	Hyperbolic functions: definition and basic properties, graphs, differentiation and integration.	The ability to sketch the graphs, differentiate and integrate hyperbolic functions. Osborn's Rule. Hyperbolic equations.
	Inverse trigonometric and hyperbolic functions and their graphs. Logarithmic form of the inverse hyperbolic functions.	Use in integration.
	The expansions of $\sin x$, $\cos x$, $\sinh x$ and $\cosh x$.	
7.	Further integration. Applications of differentiation and integration.	
	Problems on maxima and minima, curve sketching and rates of change.	
	Volume of revolution, arc length and area of surface of revolution.	The ability to calculate volume of revolution, arc length and area of surface of revolution using Cartesian or parametric coordinates. Questions will not be set on the derivation of the formulae of arc length and surface of revolution.
	Reduction formulae involving one parameter.	
8.	Polar coordinates (r,θ).	
	The relationship between Cartesian and polar coordinates. Polar curve sketching.	Students should be able to make conversions between polar and Cartesian coordinate systems. Curve sketching limited to the form $\mathbf{r} = f(\theta)$.
	Intersection of polar curves. Location of points at which tangents are parallel to, or perpendicular to, the initial line.	
	Area enclosed by a polar curve.	Use of the expression $A = \frac{1}{2}\int r^2\, d\theta$.

	Topics	Notes
9.	Elementary matrix algebra of up to 3×3 matrices. Determinants of 3×3 matrices.	Singular and non-singular matrices.
	Inverse of a non-singular 3×3 matrix.	The ability to find the inverse of a 3×3 matrix both by elementary row operations and by the adjoint method.
	Matrix representation of linear transformations in three dimensional space. Translation, rotation and reflection. Composition of transformations and matrix multiplication.	The ability to find the matrix associated with a linear transformation and vice-versa. Transformations include: rotation about a coordinate axis, reflection in the coordinate planes $x=0$, $y=0$ and $z=0$, enlargement (reduction) in which the origin is the centre of the enlargement (reduction). The ability to find the image of a point, line and plane under *any* linear transformation. Meaning of invariant points, lines and planes.
	Consideration of up to three linear equations in three unknowns, the matrix notations for the equations, solution and consistency of the equations with geometrical interpretation.	
10.	First order linear differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}+Py=Q$, where P and Q are functions of x.	
	The general solution of: $a\frac{\mathrm{d}^2x}{\mathrm{d}x^2}+b\frac{\mathrm{d}y}{\mathrm{d}x}+cy=f(x)$, where a, b and c are constants.	The function $f(x)$ will be one of the forms $p+qx$ or λe^{kx} or $p\cos nx+q\sin nx$ and the particular integral can be found by trial. [*The trial solution will be given in problems involving failure cases.*]
11.	Numerical methods: Location of roots by considering changes of sign.	
	The Newton-Raphson method.	Not more than *two* iterations will be required.
	Use of the logarithmic and exponential series, including knowledge of their ranges of convergence. Use in approximations of the first few terms of the Maclaurin series. The trapezium rule and Simpson's rule.	

	Topics	Notes
12.	Elementary probability. Calculation of probabilities of equally likely possibilities involving simple counting problems. Conditional probability. Sum and product laws. Independent events.	Permutations and combinations. The rules: $P(A') = 1 - P(A)$ $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cap B) = P(A)P(B \mid A)$